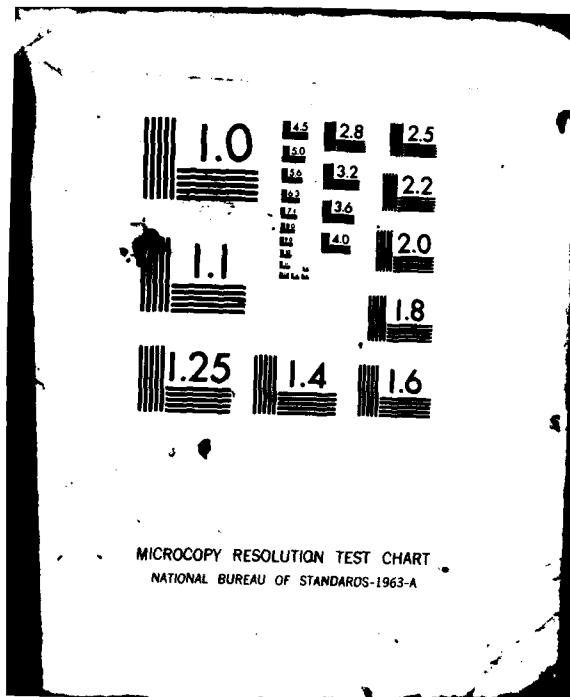


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METHODS FOR VALIDATING COMPUTER MODELS
OF MISSILE SYSTEMS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An overview of simulation model methodology is presented with emphasis on how these techniques may be applied in the missile simulation environment. Because most missile simulations produce time series output which must be matched to corresponding flight or laboratory test data, the primary focus is on time series methods. Univariate spectral analysis and cross spectral analysis techniques useful in simulation model validation are described. Examples are given using both artificially generated and real data.		

CONTENTS

	PAGE
I. INTRODUCTION	3
II. VALIDATION METHODOLOGY	3
Static Data Analysis Methods	3
Goodness of Fit Testing	4
Time Series Analysis Method	5
Internal Validity Checking	7
Screening and Preparation of Data	11
APPENDIX - COMPUTER PROGRAMS	37



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I. INTRODUCTION

Missile flight and performance characteristics are investigated using several different types of testing, including actual flight tests, laboratory tests of major system components, and computer simulation studies. For example, in addition to flight tests, extensive laboratory tests of the electronic subsystems and wind tunnel tests of the airframe are conducted. Computer simulation models may be used to gain further information about missile performance variability at a particular set of flight conditions (such as target type, speed, altitude, and maneuver; environmental factors including windspeed, temperature, and visibility, etc.), or they may be used to explore sets of conditions where no flight tests have been performed. Various types of computer simulation models have been used in analyzing missile systems, including pure digital simulators, hybrid digital-analog simulators, and hardware-in-the-loop simulators.

Model validation is an important aspect of using computer simulation as part of the test program for a missile system. By validation we mean an investigation of the consistency of the simulation model with the real missile system. Successful validation provides a basis for confidence in the model's results, and is a necessary step if the model is to be used to draw inferences about the behavior of the real missile. A reasonable definition of validity is that a set of input conditions to the model should produce output similar to that produced by the real missile system when it was exposed to the same input. Consequently, methods for comparing computer simulation model output to data generated during actual flight tests are typically used for model validation.

This paper is a review of methods useful for validation of computer simulation models of missile systems. Most of these methods discussed are statistically-based. For other more general discussions of the validation question, see Naylor and Finger (17), Van Horn (23)(24), and Kheir and Holmes (12).

II. VALIDATION METHODOLOGY

Missile performance data may be classified as either static or dynamic. Examples of static performance data are kill probabilities and terminal miss distances. Dynamic performance characteristics are output phenomena that vary continuously during missile flight, such as roll position, roll rate, wing deflection, system gain and phase, and various guidance system parameters. These characteristics are usually expressed as time series. We now briefly describe some of the more important validation techniques, and provide a list of references that discuss the procedures in more detail.

Static Data Analysis Methods - Many published discussions of simulation model validation focus on static output analysis. Standard statistical procedures, such as hypothesis testing methods, confidence intervals, and regression analysis, can be used in this context. There are also a number of statistical techniques developed especially for use in the simulation environment.

The specific statistical methodology used depends on the type of simulation model. For example, if Monte Carlo simulation is used, then by the process of replication using different random number seeds different realizations of the

output variable, say x_1, x_2, \dots, x_n can be obtained. These observations may be viewed as a random sample from some population $f(x)$ with mean μ and variance σ^2 . The sample mean and sample variance \bar{x} and s^2 are unbiased estimators of μ and σ^2 , and if the distribution $f(x)$ is not too different from the normal distribution, then relatively standard statistical methods may be used to draw inferences about these parameters. Thus if x is terminal miss distance, then an approximate 100 $(1-\alpha)$ percent confidence interval on mean terminal miss distance is

$$\bar{x} - t_{\alpha/2, n-1} s / \sqrt{n} \leq \mu \leq \bar{x} + t_{\alpha/2, n-1} s / \sqrt{n}$$

where $t_{\alpha/2, n-1}$ is the upper $\alpha/2$ percentage point of the t distribution with $n-1$ degrees of freedom. Comparison of this confidence interval with design specifications or with values observed in flight test may prove helpful in assessing the validity of the model. For an introduction to statistical methods in data analysis, see Hines and Montgomery (8), Montgomery (13), and Draper and Smith (4). An example of the use of statistical methods in simulation model validation is in Naylor, Wallace and Sasser (18).

When the simulator is nonstochastic or when replication is prohibitively expensive, statistical methods can still be helpful in parameter estimation. For example, if the simulator produces a sequence of time-oriented observations on the variable of interest, then one may obtain reasonably good estimates of the mean and variance and confidence intervals either by approximating the output as an autoregressive process and obtaining estimates of the parameters using time series methods or by breaking the output stream into k batches and treating the mean of each batch as a single observation. This latter approach is often called the method of batch means. For further details, see Fishman (5).

Goodness of Fit Testing - This approach to model validation involves testing the hypothesis that the entire sample of data generated by a computer simulation model has the same probability distribution as the sample of data observed in the flight test. Thus our attention is now focused on the conformance of the entire distribution of sample data from the simulation with the flight, and not just on the parameters of these distributions. The two-sample Kolmogorov-Smirnov test is a distribution-free test that is highly useful in this regard. For further reading on goodness of fit testing, see Conover (3).

Goodness of fit testing may often be viewed as an improvement over static data analysis methods for validation. It is entirely possible that two distributions have identical sample moments (or means and variances) but differ considerably in shape. Goodness of fit testing is designed to help detect such a situation. Its only weakness is that most goodness of fit tests require independent observations (random samples) and many computer simulations produce output streams that are highly autocorrelated. Consequently, this autocorrelation may render the goodness of fit testing approach less useful.

Time Series Analysis Method - The output variables of interest in many studies of missile systems is represented by a time series. Let x_t be the time series of interest observed in the actual flight test and y_t be the corresponding time series generated by the computer simulation model, $t=1,2,\dots,T$. These time series are usually highly autocorrelated, and may exhibit other internal structure (such as non-stationarity, the presence of deterministic components, etc.). To validate the computer simulation model, we must test the hypothesis that the two time series x_t and y_t are equivalent.

A variety of methods can be used to compare the time series x_t and y_t . In the validation of missile systems, nonstatistical methods are sometimes used. The most common method of nonstatistical comparison involves plotting the time series x_t and y_t , overlaying the plots, and sliding them along until as close a match as possible is obtained. Then the analyst determines subjectively whether or not the output time series from the simulator agrees with the flight test results. A major difficulty with this approach is that it does not quantify the risk associated with any decision, and it is entirely possible that different analysts will arrive at different conclusions.

Another nonstatistical procedure sometimes used in validating computer simulation models is Theil's inequality coefficient (19)(20)(21). This coefficient is an index which measures the conformance of one time series with another. Theil's inequality coefficient has been extensively used to validate computer simulation models of missile systems (for example, see Kheir and Holmes (12)). While this procedure is more quantitative than simple visual comparison of time series, there is no standard distribution theory for Theil's inequality coefficient, and so no statistical statements relative to the conformance of the two time series can be made.

Several statistical methods may be useful in comparing time series. One approach is to fit an appropriate stochastic model to x_t and y_t , usually an autoregressive integrated moving average model (see Box and Jenkins), (2), and then compare the two models. If the two models are the same, the inference is that the two time series are the same. A test for the equivalence of two time series models is described by Hsu and Hunter (10), who also illustrate the use of the procedure in validating a computer simulation model of an airport. Unfortunately, the two time series could have been generated by the same underlying stochastic model and still differ significantly in certain characteristics, particularly over the relatively short records typically associated with time series obtained from missile systems. For example, the two series could be significantly out of phase, and yet both could have been generated from the same AR(2) model (say). Furthermore, differences in phase angle, gain and frequency usually have specific interpretations to the missile designer. Therefore, he would like to know if such differences are present.

Spectral methods have been suggested by many authors for validation of computer simulation models (6)(14)(18). The general approach consists of comparing the sample spectra of the simulation model output and the corresponding flight test data to infer how well the simulation matches the flight.

The spectrum of a time series x_t , say $\phi_{xx}(\omega)$, is a decomposition of the total variance of the series by frequency over the interval $0 \leq \omega \leq \pi$. Thus $\phi_{xx}(\omega)$ measures the variance contribution to x_t at frequency ω . The spectrum is

related to the autocovariance function of a wide-sense stationary time series by the relationship

$$\phi_{xx}(\omega) = \gamma_0 + 2 \sum_{k=1}^{\infty} \gamma_k \cos(\omega k) \quad (1)$$

where $\{\gamma_k\}, k=0,1,2,\dots$ is the autocovariance function of the time series x_t . Thus the spectrum is the Fourier cosine transform of the autocovariance function. The spectrum is estimated by the sample spectrum

$$f_{xx}(\omega_j) = \gamma_0 c_0 + 2 \sum_{k=1}^m \lambda_k c_k \cos(\omega_j k) \quad (2)$$

where $f_{xx}(\omega_j)$ is an estimate of the spectrum averaged over a band of frequencies centered at $\omega_j = \pi j/m$, $j=0,1,\dots,m$, m is the number of frequency bands estimated, λ_k , $k=0,1,\dots,m$ are a set of constants or weights, and

$$c_k = \frac{1}{T-k} \sum_{t=1}^{T-k} (x_t - \bar{x})(x_{t+k} - \bar{x}), \quad k=1,2,\dots,m$$

is the sample autocovariance function. The weights $\{\lambda_k\}$ depend on the type of spectral window used in the estimation process (see Fuller (7) and Jenkins and Watts (11)). Spectral windows are employed to give a smoother estimate of the spectrum.

Let $f_{xx}(\omega_j)$ denote the sample spectrum of the flight test data and $f_{yy}(\omega_j)$ denote the sample spectrum of the simulation output. To compare these spectra at a specific frequency construct a $100(1-\alpha)$ percent confidence interval on the ratio of the true spectra, say $\phi_{xx}(\omega_j)$, using

$$\frac{f_{xx}(\omega_j)/f_{yy}(\omega_j)}{F_{\alpha/2,k,k}} \leq \phi_{xx}(\omega_j)/\phi_{yy}(\omega_j) \leq \frac{f_{xx}(\omega_j)/f_{yy}(\omega_j)}{F_{1-\alpha/2,k,k}} \quad (3)$$

$$j=0,1,2,\dots,m$$

where $F_{p,k,k}$ is the p th percentage point of the F distribution with $k = 2T/m$ degrees of freedom in the numerator and denominator. This succession of confidence intervals at the frequency points ω_j , $j=0,1,\dots,m$ is called a confidence band. If the upper and lower confidence limits contain the value $\phi_1(\omega_j)/\phi_2(\omega_j) = 1$, then we conclude that at that frequency the two time series are identical.

For the time series to be identical, their spectra must be equal at all frequencies ω_j , $j=0,1,\dots,m$. The simultaneous confidence band allows us to state with a probability at least $1-\alpha$ that all $m+1$ confidence intervals are simultaneously true. The $100(1-\alpha)$ percent simultaneous confidence band is computed from

For example, if we wished to make the statement that all $m+1$ frequencies are simultaneously equal with a probability of at most 0.05 (that is, a 95 percent simultaneous confidence interval), then the probability level associated with the F -value at each frequency is $0.05/2(m+1) = 0.025/(m+1)$. Thus if there are 16 bands in the sample spectrum, then

$$\frac{0.025}{16} = 0.00156$$

probability to each tail of the F distribution. Note that the simultaneous confidence band (4) is wider than the single confidence interval (3) at any frequency. For an application of this methodology to the validation of a computer simulation model of a missile system, see Montgomery and Conard (14).

In constructing confidence intervals, the analyst must specify α , the Type I error rate. In other words, α is the probability that we will conclude that the simulation and flight-test data differ when they really do not. A reasonable range of values is $0.01 \leq \alpha \leq 0.10$. Values of $\alpha > 0.1$ imply that it is relatively easy to find that the flight test and the simulation differ when they really do not, while values of $\alpha < 0.01$ imply that it is easy to conclude that the flight test and the simulation match when they really do not. Note that the confidence intervals (3) and (4) become wider as α becomes smaller. Thus very small values of α make it easier to conclude that the two time series agree.

Spectral methods can be applied only to a stationary series. If the series is nonstationary, then the nonstationary part of the process must be removed, either by successive differencing or by fitting a polynomial model (or other appropriate function) to the data and analyzing the residuals. Piecewise polynomial fitting may be necessary when the time series exhibits different behavior in different local segments of time. This may be conveniently done using splines. Several useful references are in Draper and Smith (4). Indications of nonstationarity are usually observed in either the sample autocorrelation function or the sample spectrum. If the autocorrelation function does not die down even at very long lags or if the power is concentrated at the lowest frequency in the spectrum, then the series is probably nonstationary. Note that if two nonstationary series x_t and y_t are compared, they are equivalent in a frequency sense if both their stationary representations have the same spectrum and if the same level of differencing (or the same order polynomial) is required to reduce both of them to stationarity.

Internal Validity Checking - While simulation-to-flight test comparisons form the basis of simulation model validation, it is also usually necessary to validate the internal logic of the simulation model. Spectral methods can be useful in this aspect of validation also. These techniques can be used to investigate the interrelationships between two time series generated by the simulator. For example, suppose that the computer simulation model produces time series output

of fin deflection and airframe lateral acceleration. The lateral accelerations are a physical result of the fin deflections and the nonlinear aerodynamic response of the airframe. Causal or correlative structure between these two time series should be reflected in the analysis.

Interrelationships between two time series x_t and y_t are based on the cross-spectrum. The cross-covariance function is

$$\gamma_{xy}(k) = E(x_t - \mu_x)(y_{t+k} - \mu_y) \quad (5)$$

where x and y are the means of x_t and y_t respectively. Unlike the autocorrelation function for a single time series, the cross-correlation function $\gamma_{xy}(k)$ may not be symmetric about zero. The cross-spectrum is defined as

$$\phi_{xy}(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_{xy}(k) e^{-i\omega k} \quad (6)$$

Note that $\phi_{xy}(\omega)$ is a continuous periodic function of ω (the frequency). Since $\gamma_{xy}(k)$ may not be symmetric about zero, the cross-spectrum is in general a complex function, say

$$\phi_{xy}(\omega) = C_{xy}(\omega) - i q_{xy}(\omega), \quad (7)$$

where $C_{xy}(\omega)$ is the coincident spectral density (cospectrum) and $q_{xy}(\omega)$ is the quadrature spectral density. Both $C_{xy}(\omega)$ and $q_{xy}(\omega)$ are real-valued functions of ω . $C_{xy}(\omega)$ is the cosine portion of the transform and is an even function of ω , while $q_{xy}(\omega)$ is the sine portion of the transform and is an odd function of ω . Let $\hat{\phi}_{xy}(\omega)$ denote an estimate of the cross-spectrum and, let $\hat{C}_{xy}(\omega)$ and $\hat{q}_{xy}(\omega)$ denote the estimates of $C_{xy}(\omega)$ and $q_{xy}(\omega)$ respectively. For an introduction to the estimation problem see Fuller (7) and Jenkins and Watts (11).

The squared coherency is defined as

$$K_{xy}^2(\omega) = \frac{|\phi_{xy}(\omega)|^2}{\phi_{xx}(\omega) \cdot \phi_{yy}(\omega)} \quad (8)$$

where

$$|\phi_{xy}(\omega)|^2 = C_{xy}^2(\omega) + q_{xy}^2(\omega),$$

and $\phi_{xx}(\omega)$ and $\phi_{yy}(\omega)$ are the spectra of x_t and y_t , respectively. The coherency is analogous to the coefficient of multiple determination R^2 in multiple regression. Thus coherency is a measure of independence of x_t and y_t at frequency ω . If coherency is 0, then the series are independent (unrelated), while if coherency is 1, then the series are perfectly dependent (related). Coherency is a nondimensional measure of the correlation between two time series as a function of frequency. Another way to think of coherency is in terms of the predictability of one series from the other. If coherency is 0, then one series cannot be predicted from the other, while if coherency is 1, one series can be perfectly predicted from the other. We may also think of coherency as the proportion of the total power (by frequency) in one time series that can be explained by the other time series.

An F-test for zero coherency is given by

$$F_0 = \frac{4d \hat{K}_{xy}^2(\omega)}{2[1 - \hat{K}_{xy}^2(\omega)]} \quad (9)$$

which if $\hat{K}_{xy}^2(\omega) = 0$ is distributed as $F_{2,4d}$, where d is the number of points at which the spectrum is estimated, and

$$\hat{K}_{xy}^2(\omega) = \frac{|f_{xy}(\omega)|^2}{\hat{f}_{xx}(\omega) f_{yy}(\omega)} \quad (10)$$

is an estimate of coherency. If $F_0 > F_{2,4d}$, the hypothesis of zero coherency is rejected. While the limiting values of coherency, 0 and 1, are of obvious interest, intermediate values are also of interest because of the natural interpretation of coherency as a "percent variability explained". If the coherency function is greater than zero but less than unity, one or more of three possibilities exist

1. Extraneous noise is present in the measurements,
2. The system relating x_t and y_t is not linear, or
3. y_t is an output related to the input x_t as well as to other inputs.

The gain of y_t over x_t is defined as

$$G_{xy}(\omega) = \frac{A_{xy}(\omega)}{f_{xx}(\omega)} = \frac{[C_{xy}^2(\omega) + Q_{xy}^2(\omega)]^{1/2}}{\phi_{xx}(\omega)} \quad (11)$$

The gain behaves like a regression coefficient in the regression of y on x through the origin, but it is now evaluated at frequency ω . That is, gain measures the increase in amplitude of y_t over that of x_t at frequency ω . One may construct a $100(1-\alpha)$ percent confidence interval on gain, using

$$\max\{0, \hat{G}_{xy}(\omega) - \Delta\} \leq G_{xy}(\omega) \leq \hat{G}_{xy}(\omega) + \Delta \quad (12)$$

where

$$\Delta = [(2d + 1)^{-1} f_{xx}(\omega) f_{zz}(\omega) F_{\alpha/2, 4d}]^{1/2}$$

$$\hat{G}_{xy}(\omega) = \frac{\hat{A}_{xy}(\omega)}{f_{xx}(\omega)} = \frac{[\hat{C}_{xy}^2(\omega) + q_{xy}^2(\omega)]^{1/2}}{f_{xx}(\omega)}$$

and

$$f_{zz}(\omega) = f_{yy}(\omega) [1 - \hat{K}_{xy}^2(\omega)]^{2d+1}$$

The phase spectrum is defined as

$$\phi_{xy}(\omega) = \tan^{-1} [-q_{xy}(\omega)/C_{xy}(\omega)] \quad (13)$$

and estimated by

$$\hat{\phi}_{xy}(\omega) = \tan^{-1} [-\hat{q}_{xy}(\omega)/\hat{C}_{xy}(\omega)]. \quad (14)$$

The phase spectrum shows whether the frequency components in one series lead or lag the components at the same frequency in the other series. If the coherency is zero at frequency ω , that implies that a $100(1-\alpha)$ percent confidence interval on the phase angle is $(-\pi/2, \pi/2)$. That is, the average phase difference between the two processes is zero, but the phase difference is equally likely to lie anywhere in the range $(-\pi/2, \pi/2)$. If coherency is not zero, then a $100(1-\alpha)$ percent confidence interval is

$$\hat{\phi}_{xy}(\omega) - \delta \leq \phi_{xy}(\omega) \leq \hat{\phi}_{xy}(\omega) + \delta, \quad (15)$$

where

$$\delta = \sin^{-1} \left[t_{\alpha, 4d}^2 \frac{1 - \hat{K}_{xy}^2(\omega)}{4d \hat{K}_{xy}^2(\omega)} \right]^{1/2}.$$

Generally speaking, the cross correlation structure between two time series can be adequately described by their squared coherency and phase spectra. Therefore, it is recommended that internal validity checking concentrate on these measures.

Screening and Preparation of Data - A potentially frustrating problem for the data analyst is dealing with wild or unusual observations in either the observed flight data or the simulation data. These wild or aberrant observations may severely distort the sample spectrum or estimates of the parameters of the underlying distributions. Often we find that some type of data editing or preliminary screening of the data is necessary.

Two approaches are useful in this regard. The first of these is to smooth the data with a nonlinear robust filter to eliminate spiky noise. The more popular nonlinear smoothers are usually based on running medians (see Tukey (22) and Velleman (25)). A second approach is to fit a model to the data that describes the smooth portion of the signal. Fitting methods more robust than least squares are recommended in this approach. It is well-known that least squares is severely distorted by outliers. Robust fitting procedures are discussed by Hogg (9). Agee and Turner (1) describe the use of these methods in preprocessing of missile trajectory data.

Examples - The validation and analysis methodology presented in the previous sections has been implemented with a FORTRAN IV computer program. This program and its operation are described in more detail in the Appendix. In this section we will illustrate the use of the methodology with several examples.

Examples Using Simulated Data - In order to check program operation and investigate the effectiveness of the proposed methods, sample time series were generated from stochastic processes having known spectral properties. The ability of the procedure to produce sample spectral estimates matching the population values reasonably closely is then investigated.

A plot of two realizations of length 200 from the AR(2) process

$$x_t = x_{t-1} - 0.5x_{t-2} + a_t$$

$$a_t \sim NID(0, \sigma_a^2)$$

is shown in Figure 1. The theoretical autocorrelation function and spectrum are shown in Figure 2. The sample autocorrelation functions for both realizations are shown in Figures 3 and 4. Both plots exhibit the characteristic pseudocyclic decay associated with the AR(2) process. Figures 5 and 6 present the smoothed spectral estimates for truncation points of $m = 8, 16$, and 32 . At $m = 8$, the low-frequency power in the series is easily seen, but there is no indication of the peak. Increasing m to 16 gives some indication of the peak for both realizations. At $m = 32$, other peaks occur because of the increase in variance of the estimate. While it is difficult to obtain a good estimate of a narrow two-sided peak with a realization of length 200, we conclude that $m = 16$ or perhaps $m = 24$ would be adequate to show the major features of the spectrum.

A comparison of the two spectra using equations (3) and (4) with a truncation point $m = 24$ is shown in Table 1. Individual 95 percent confidence limits on the ratio of the spectra are shown in columns (e) and (f), while the corresponding simultaneous confidence limits are shown in columns (g) and (h). The simultaneous confidence limits indicate agreement between the spectra at 21 of the 25 tabulated points, and only one of the four mismatched points corresponds to a frequency in the spectrum having significant power. Considering the difficulty in estimating a spectrum with a narrow two-sided peak, we conclude that there is no strong evidence that the two series are not realizations of the same underlying stochastic process.

The second simulated example consists of 100 realizations of the bivariate linear process

$$x_t = 0.6x_{t-1} - 0.5y_{t-1} + a_t$$

$$y_t = 0.4x_{t-1} + 0.5y_{t-1} + b_t$$

$$a_t \sim NID(0,1), b_t \sim NID(0,1)$$

described in Jenkins and Watts (11). Both series are plotted in Figure 7. The sample autocorrelation functions and spectra (with truncation points $m = 8, 16$ and 32) for x_t and y_t are shown in Figures 8, 9, 10, and 11, respectively. Note that although the bivariate process is first order, the autocorrelation function and spectrum resemble those of a second-order univariate process. Furthermore, both series have the same general behavior, with a peak or trough in x_t followed by a corresponding peak or trough in y_t after one or two observations. This is confirmed from inspection of the sample cross correlation function shown in Figure 12, which shows larger spikes at positive lags 1 and 2.

The comparison of the univariate spectra is shown in Table 2, based on the truncation point $m = 16$, and with a significant level of 95 percent. Inspection of the simultaneous confidence limits in columns (g) and (h) reveal that the two series closely agree.

Plot of original series

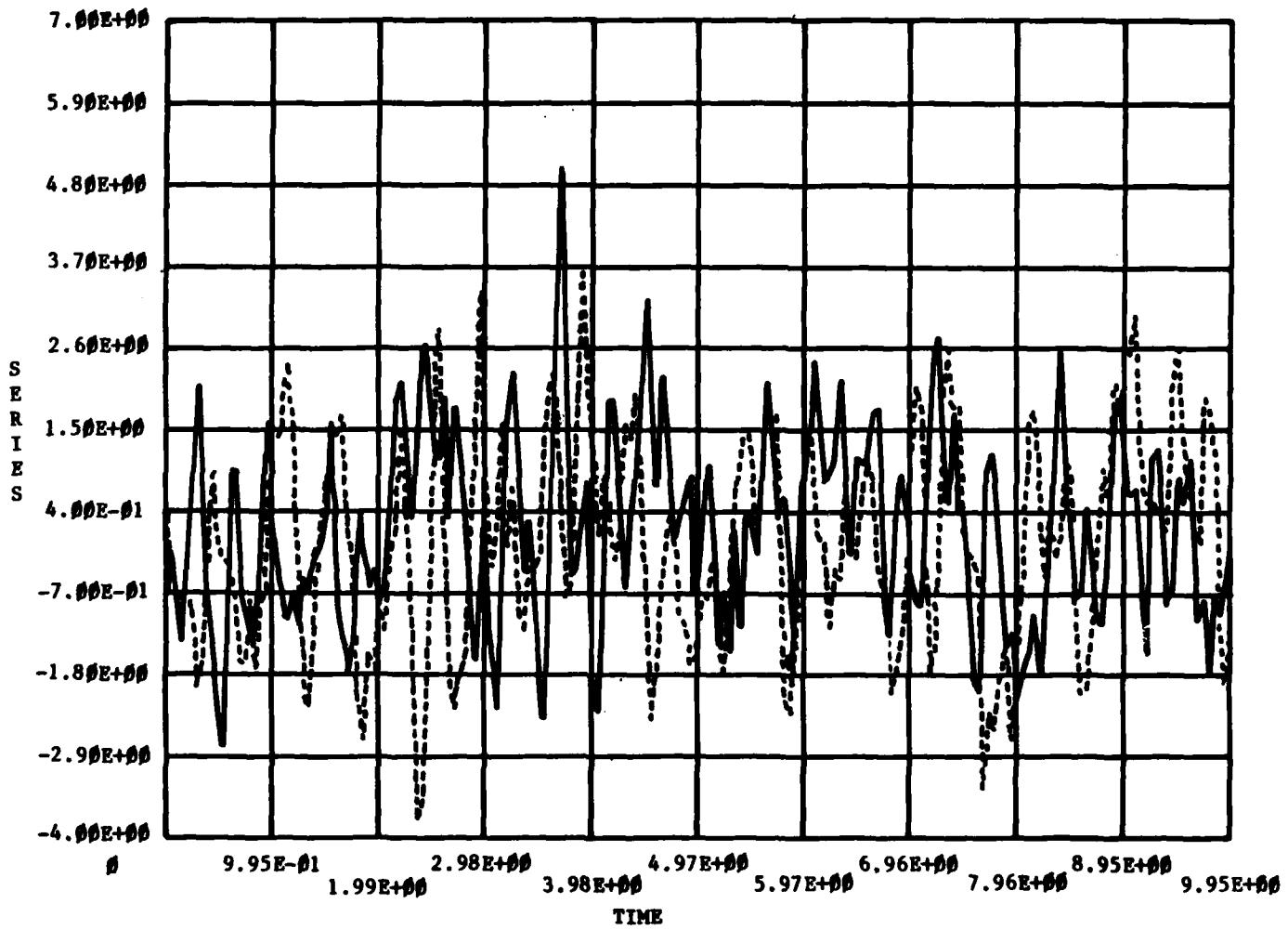


Figure 1. Two Realizations of the AR(2) Process $x_t = x_{t-1} - 0.5x_{t-2} + \epsilon_t$.

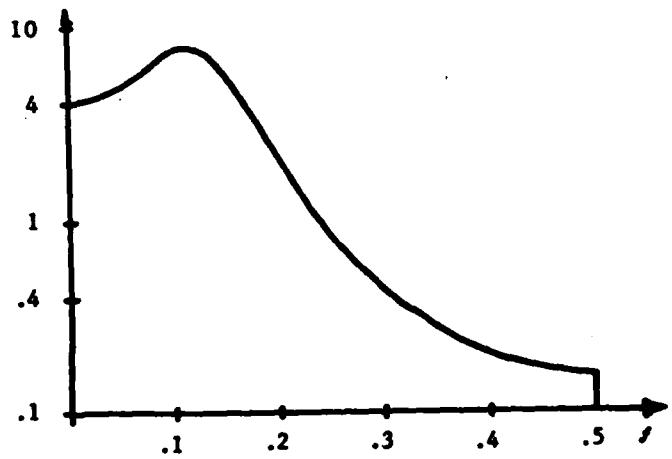
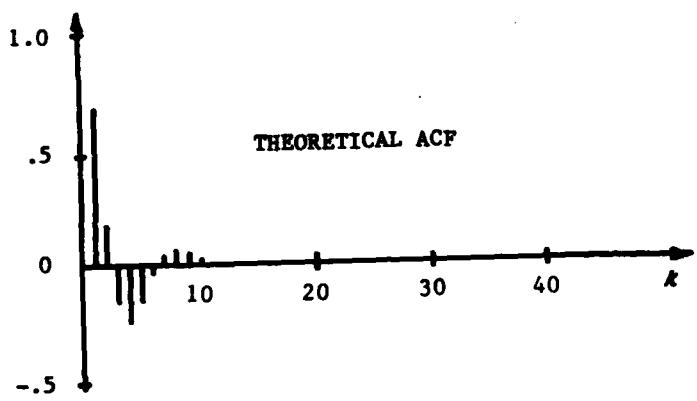


Figure 2. Theoretical Autocorrelation Function and Spectrum for the AR(2) Process $x_t = x_{t-1} - 0.5x_{t-2} + a_t$

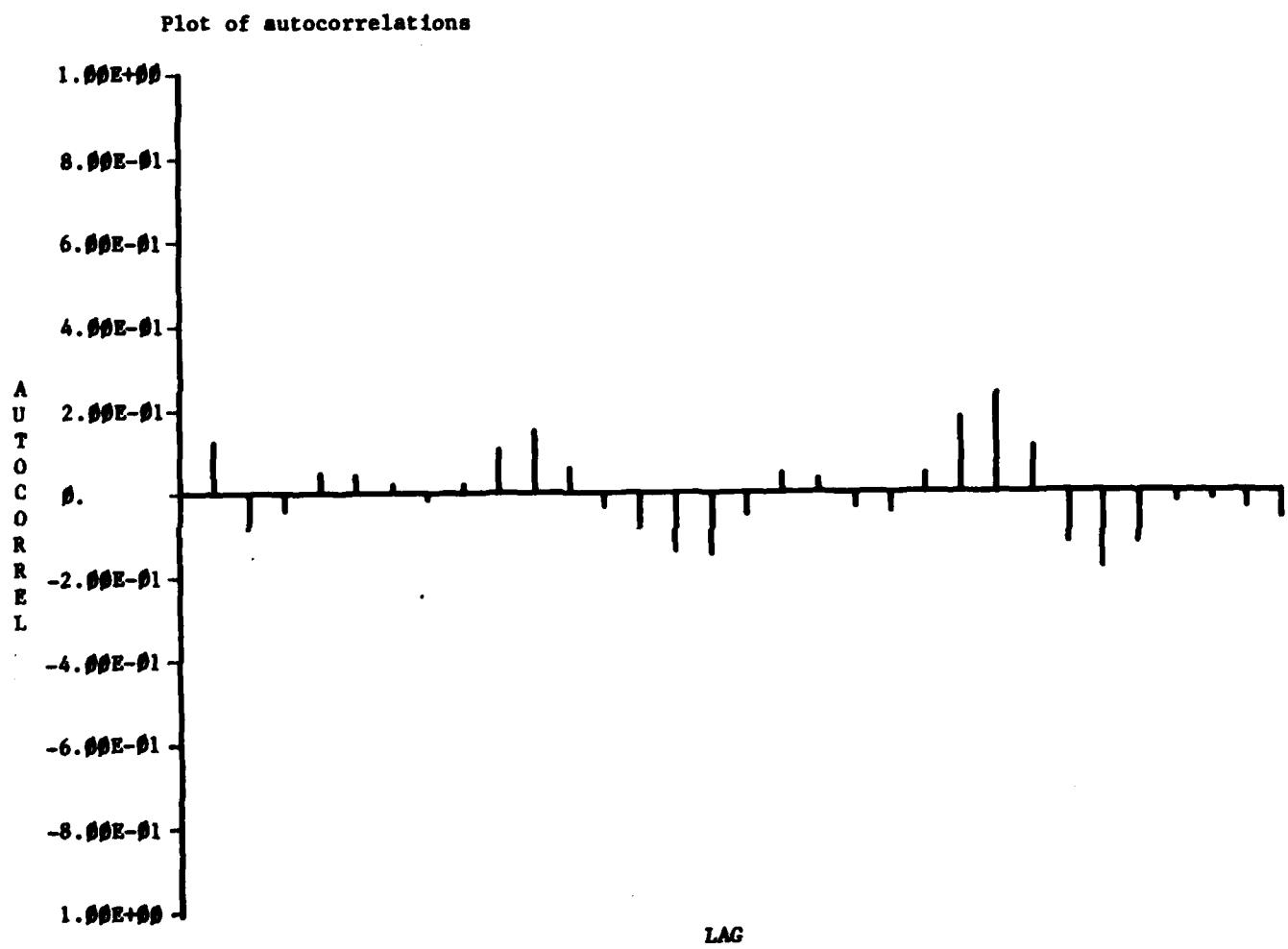


Figure 3. Sample ACF of Realization one, x_t .

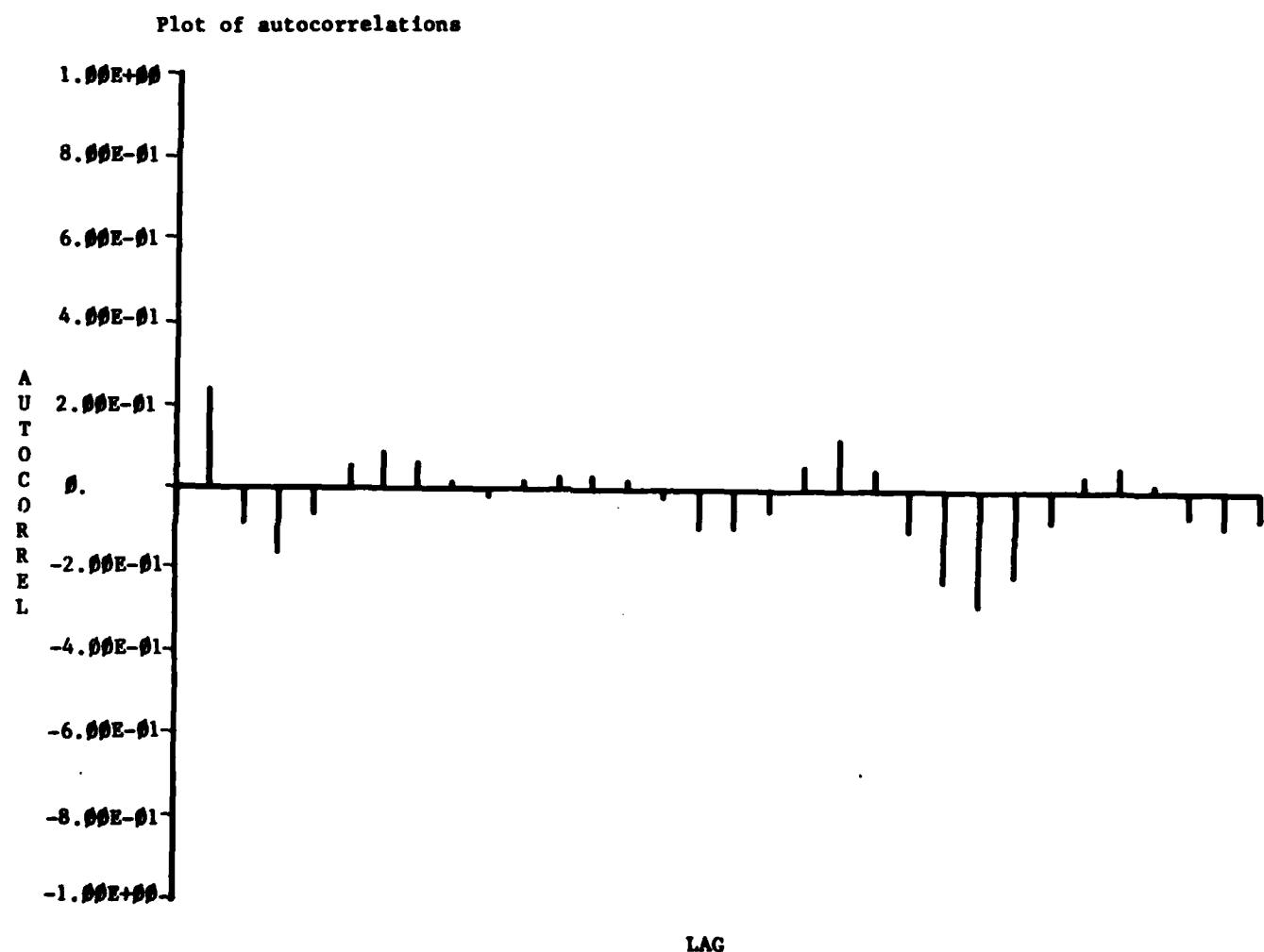


Figure 4. Sample ACF of Realization 2, y_t .

Spectrum of series 1 and 2

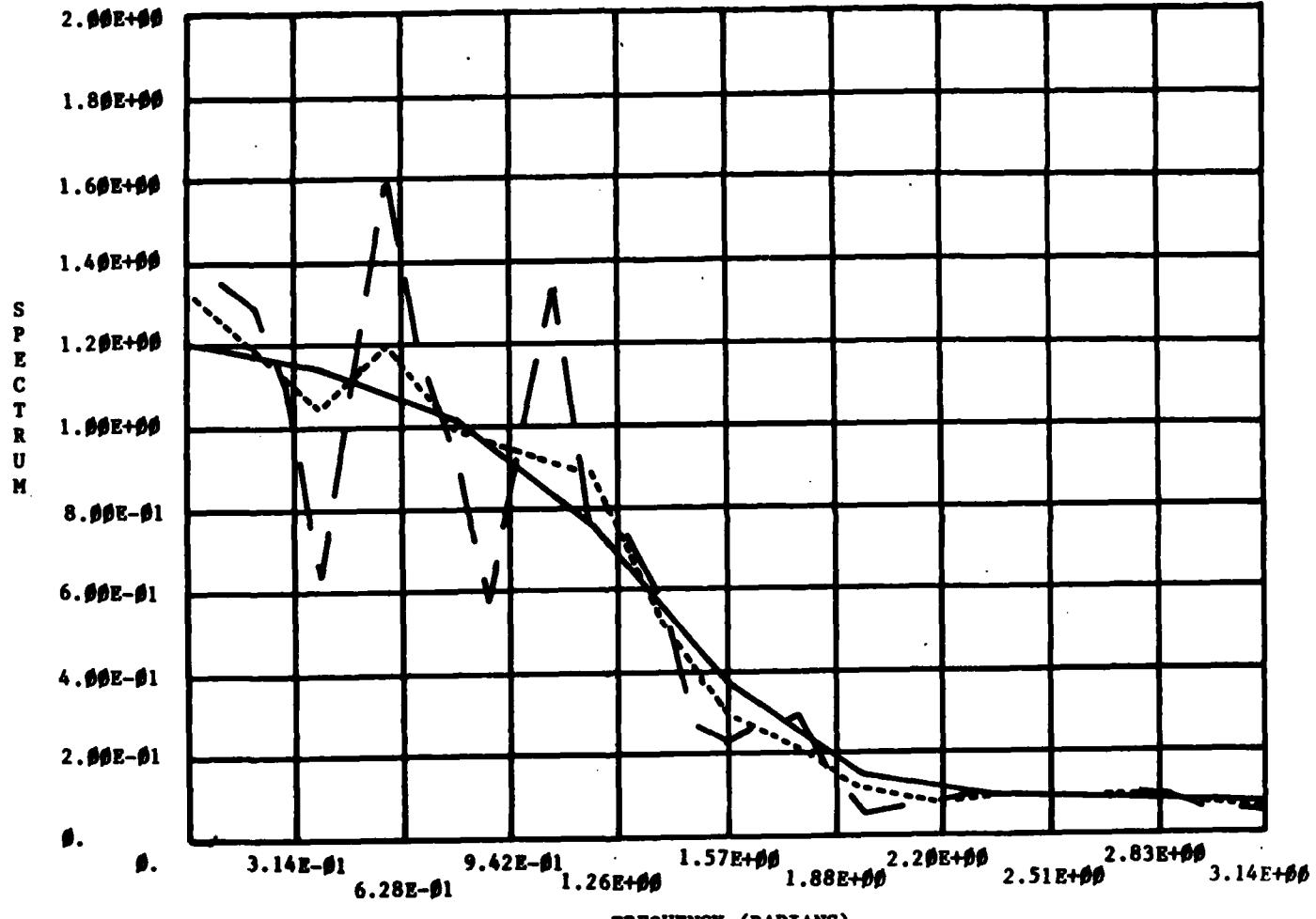


Figure 5. Spectral Estimates for x_t .

18
Spectrum of series 1 and 2

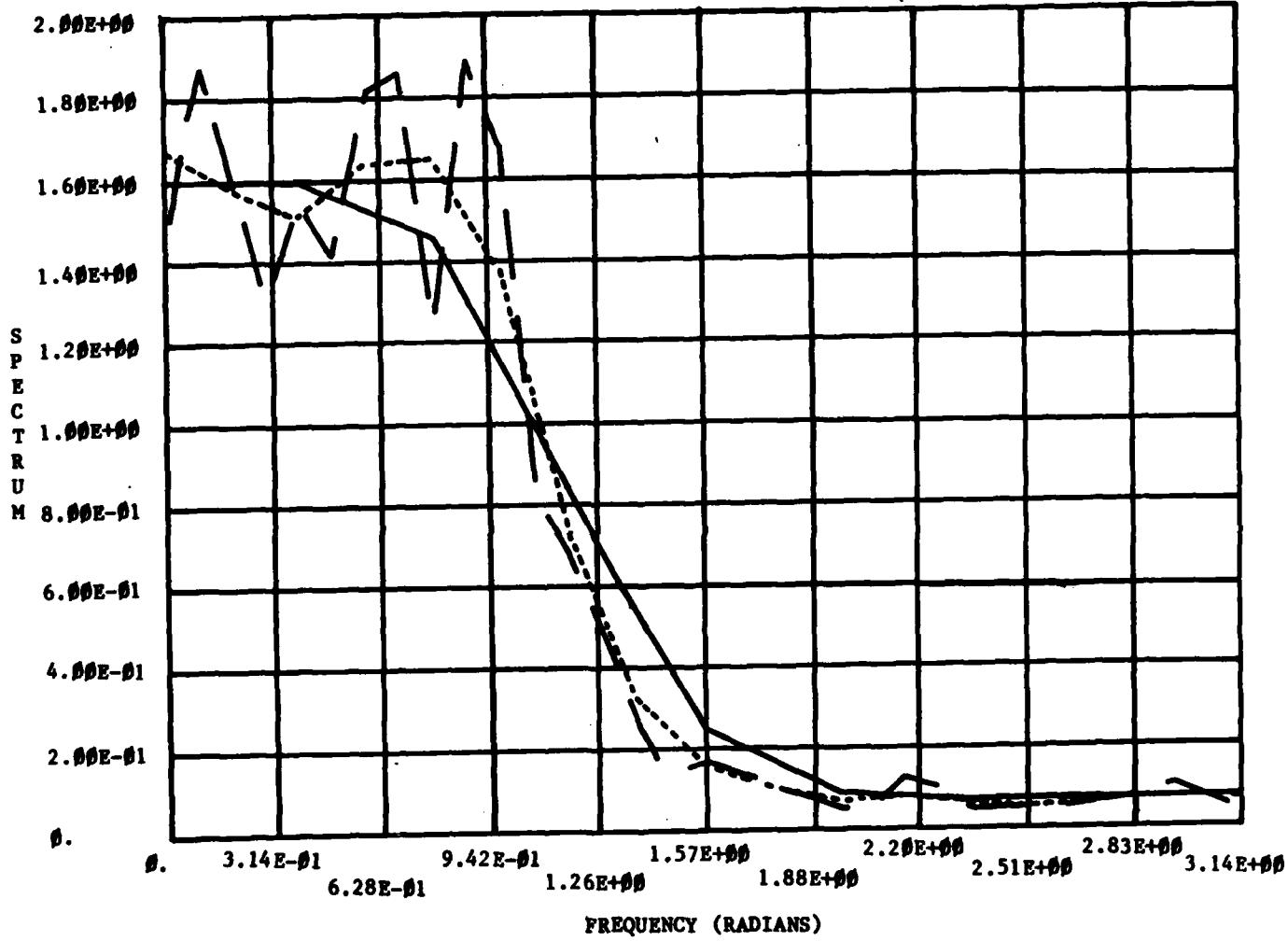


Figure 6. Spectral Estimates for y_t .

Plot of original series

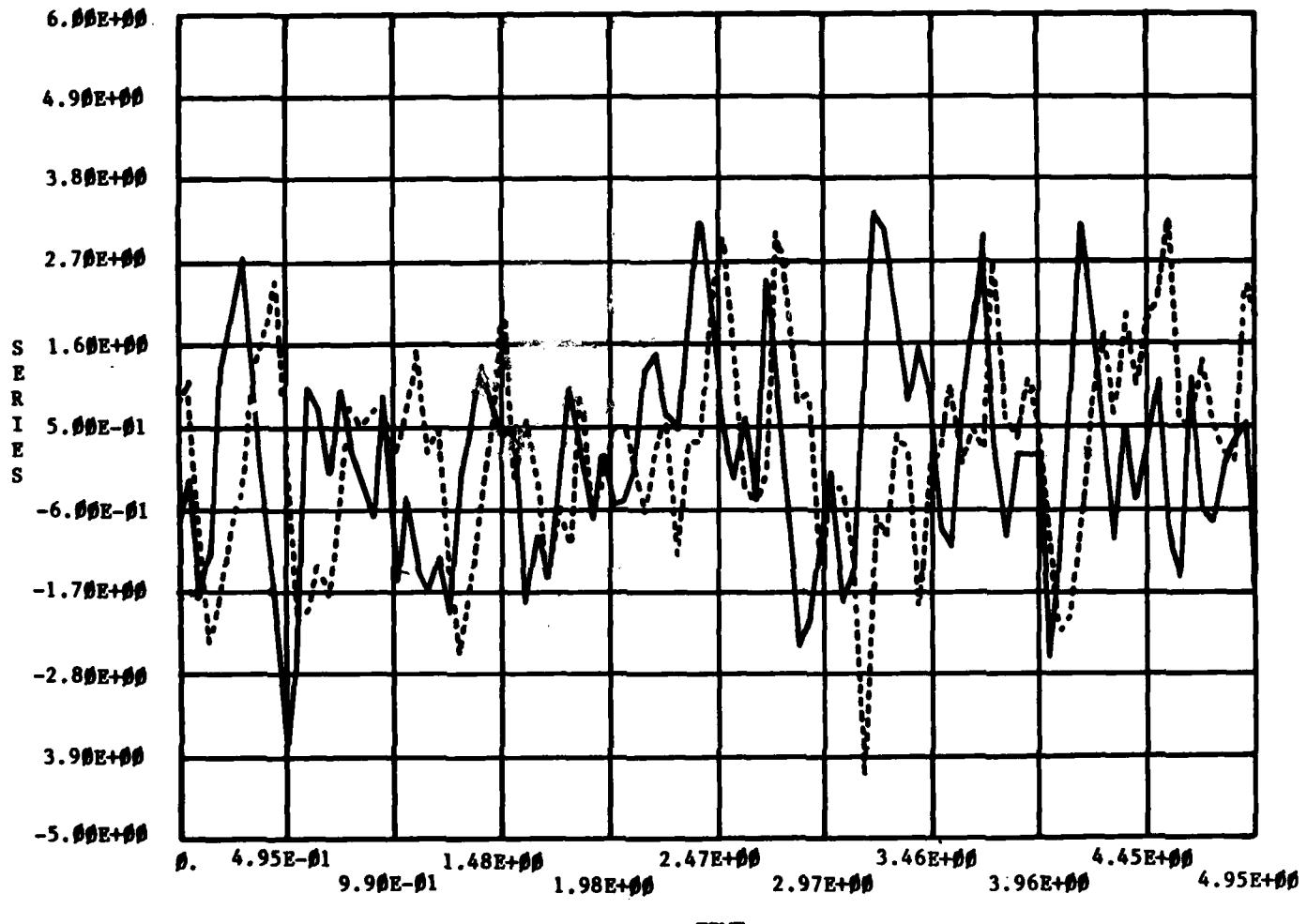


Figure 7. Realization of the Bivariate Process $x_t = 0.6x_{t-1} - 0.5y_{t-1} + a_t$,
 $y_t = 0.4x_{t-1} + 0.5y_{t-1} + b_t$.

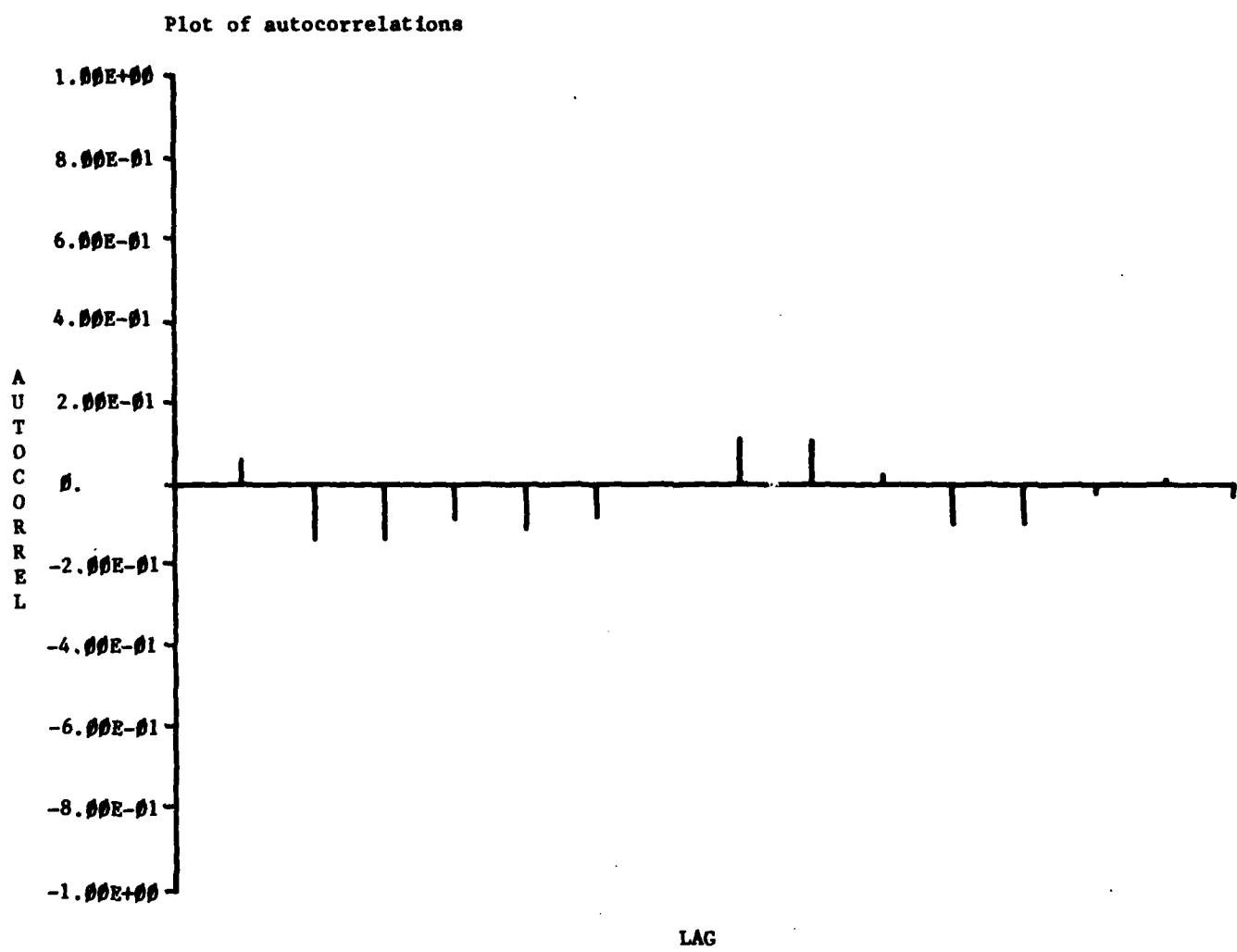


Figure 8. Sample ACF, x_t .

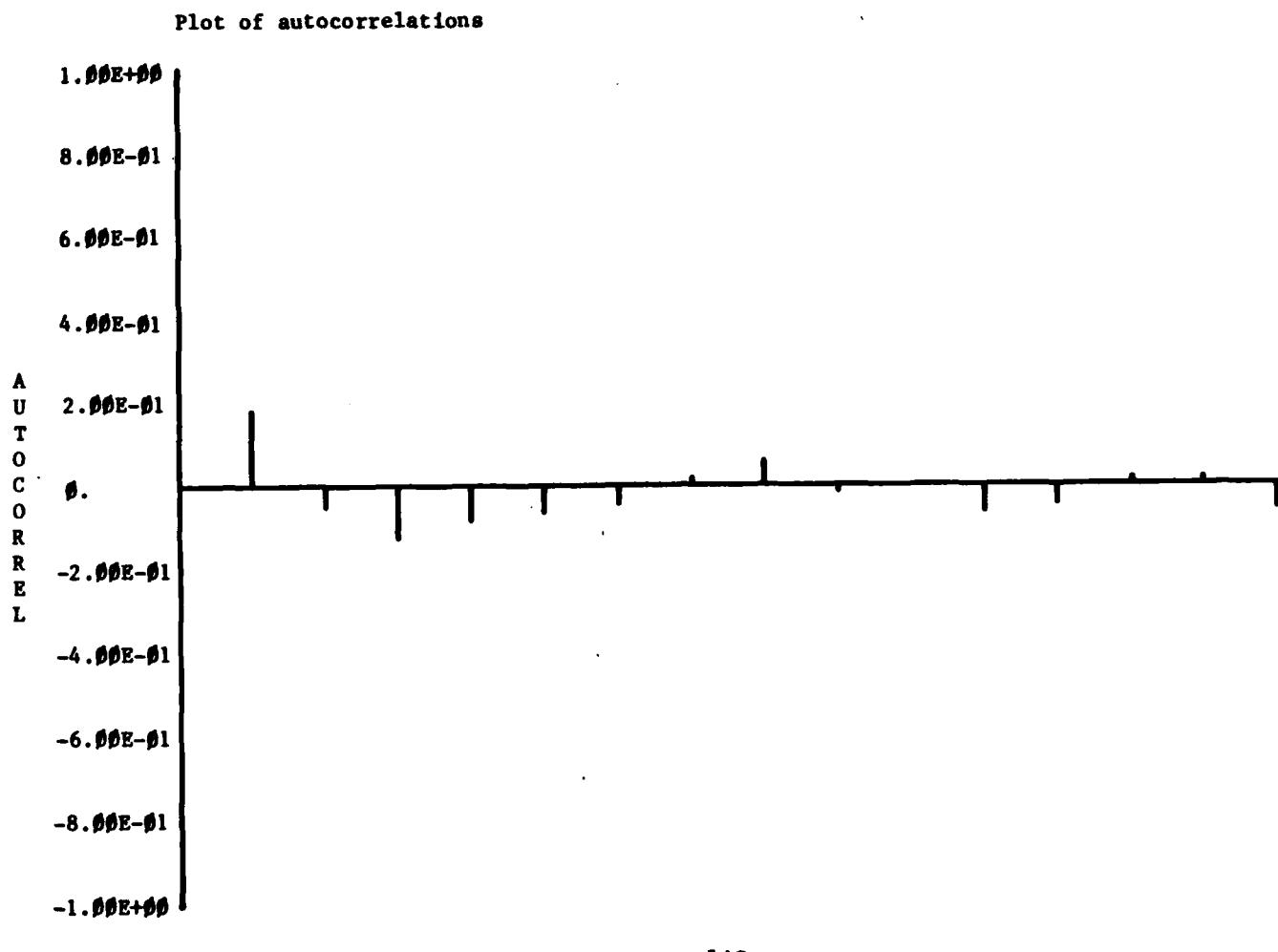
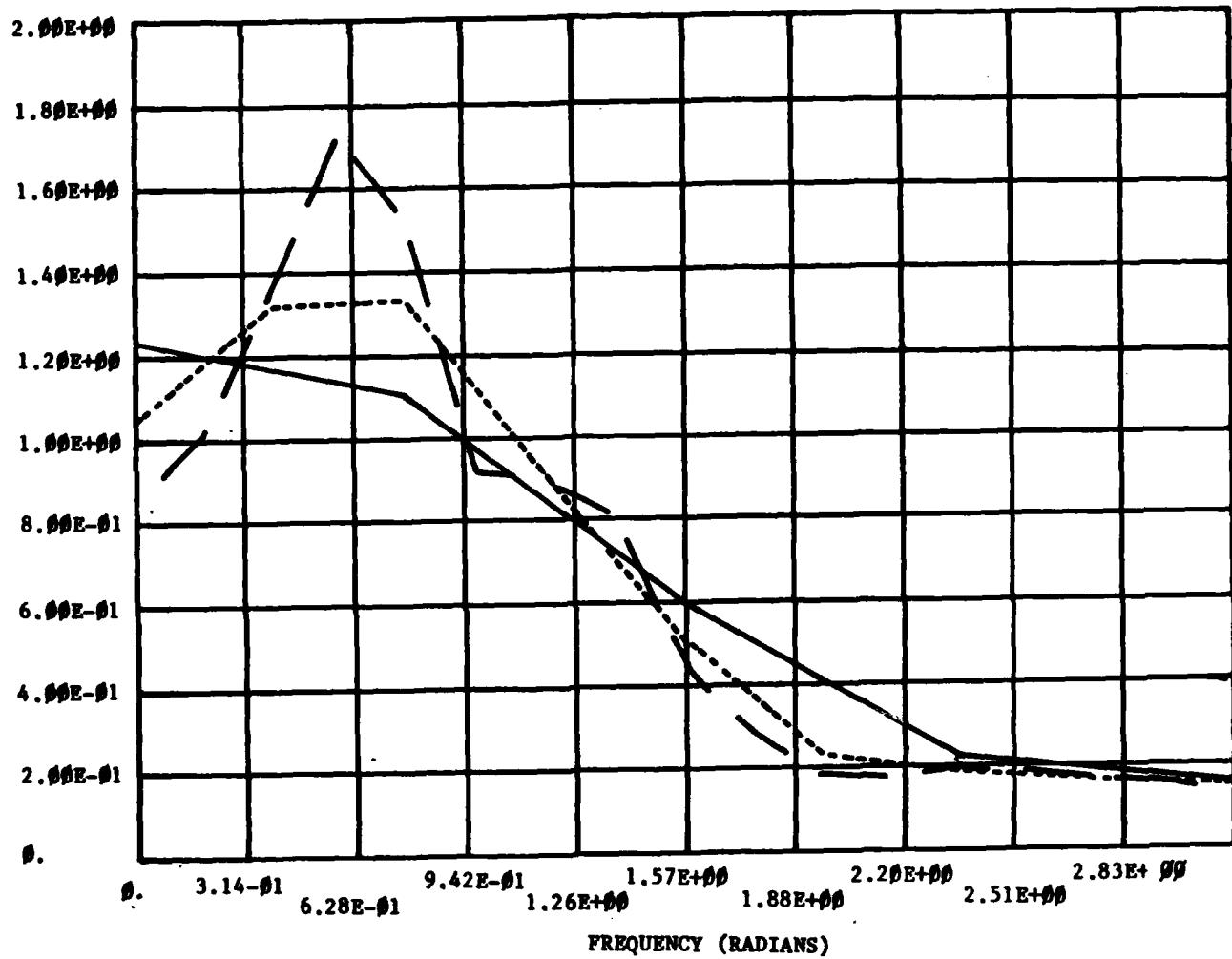


Figure 9. Sample ACF, y_t .

Spectrum of series 1 and 2

Figure 10. Sample Spectrum for x_t with $m = 8, 16$, and 32 .

Spectrum of series 1 and 2

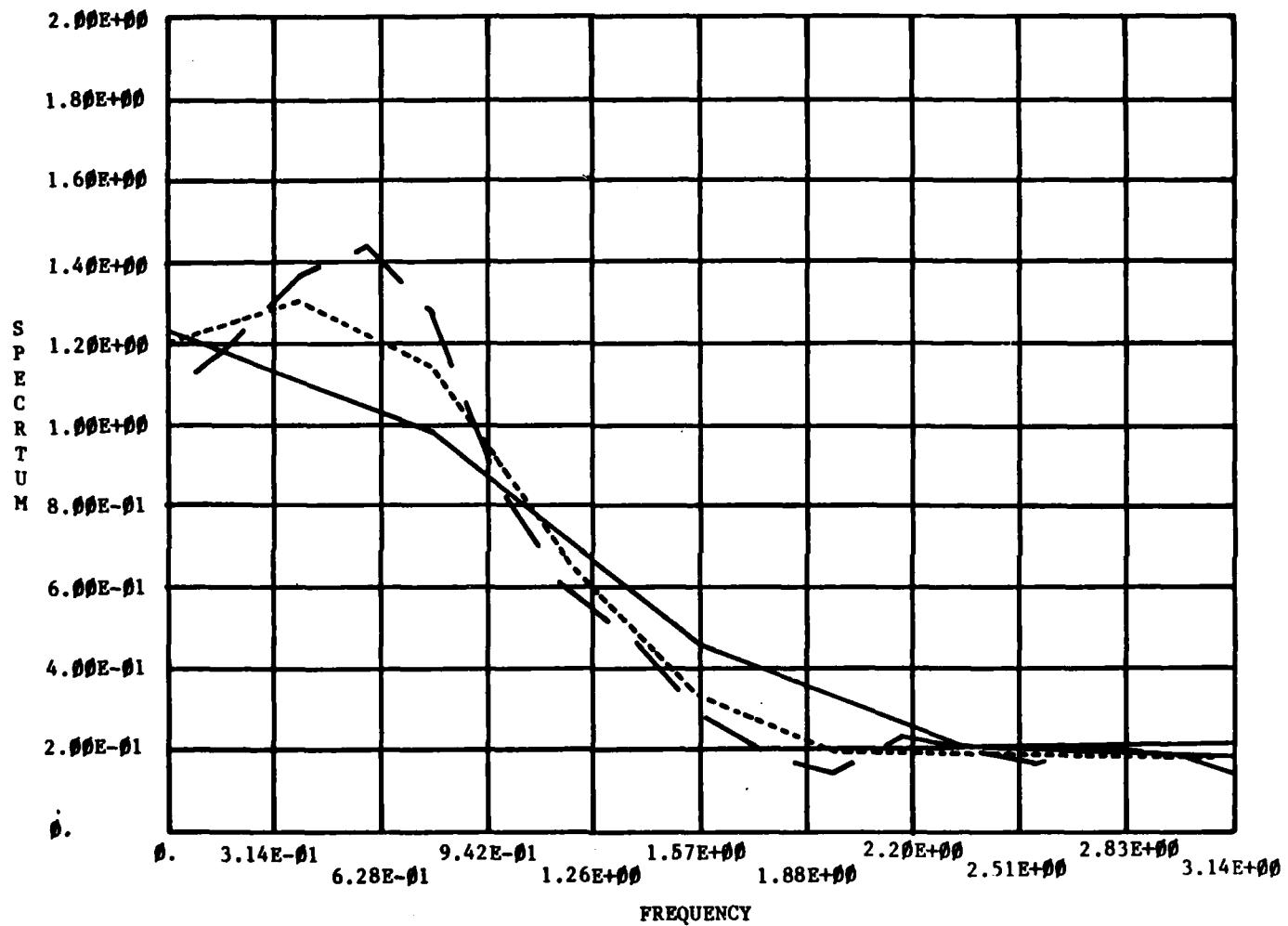


Figure 11. Sample Spectrum for y_t with $m = 8, 16$, and 32 .

Plot of cross correlation

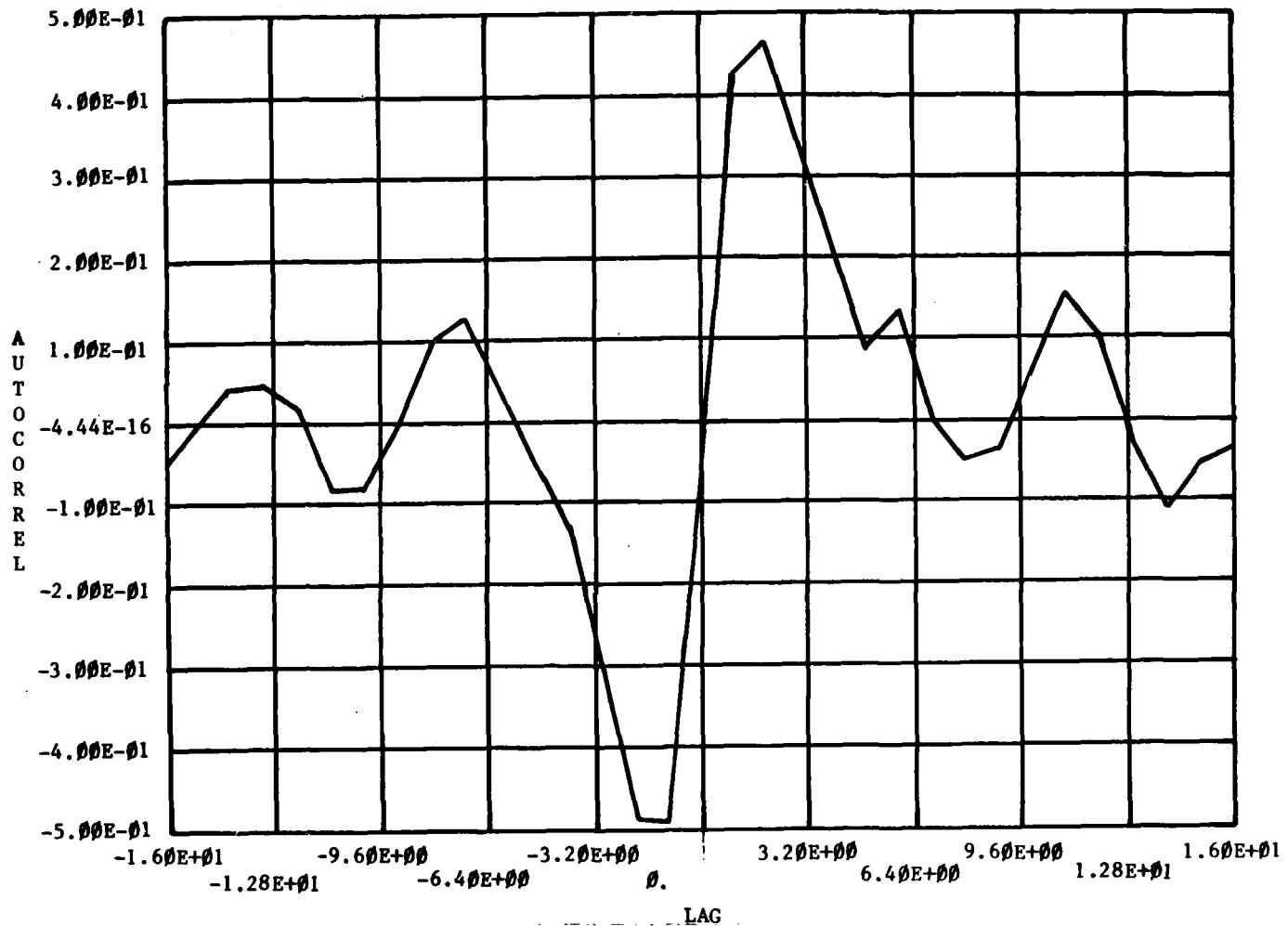


Figure 12. Sample Cross Correlation Function

TABLE 1
Comparison of Spectra

(a) FREQ	(b) X SPECTRUM	(c) Y SPECTRUM	(d) RATIO	(e) INDIV CI LB	(f) INDIV CI UB	(g) SIMU CI LB	(h) SIMU CI UB
0.000000	1.361527	1.644083	.828137	.553752	1.238480	.437674	1.566946
.020833	1.326784	1.713186	.774454	.517856	1.158197	.409302	1.465369
.041667	1.133410	1.475656	.768072	.513588	1.148652	.405929	1.453293
.062500	.838198	1.453755	.576575	.385539	.862268	.304722	1.090956
.083333	1.268663	1.603301	.786292	.525772	1.175901	.415559	1.487769
.184167	1.322648	1.762093	.750612	.501913	1.122541	.396702	1.420257
.125000	.889412	1.543782	.576126	.385239	.861597	.304485	1.090106
.145833	.789594	1.718659	.459425	.307204	.687070	.242808	.869292
.166667	1.125775	1.173977	.958941	.641217	1.434097	.506804	1.814443
.187500	.899432	.660522	1.361699	.910530	2.036423	.719664	2.576515
.208333	.656709	.415672	1.579871	1.056416	2.362699	.834968	2.989325
.229167	.401155	.196577	2.035109	1.360821	3.043508	1.075564	3.850696
.250000	.236488	.155779	1.517973	1.015026	2.270131	.802255	2.872206
.270833	.272040	.128775	2.112519	1.412583	3.159275	1.116475	3.997166
.291667	.192898	.090467	2.132247	1.425774	3.188778	1.126901	4.034494
.312500	.076704	.056107	1.367091	.914135	2.044486	.722513	2.586717
.333333	.067223	.079530	.845254	.565198	1.264078	.446720	1.599332
.354167	.089081	.100406	.887214	.593255	1.326830	.468896	1.678727
.375000	.089630	.056462	1.587441	1.061478	2.374021	.838969	3.003650
.395833	.088248	.046995	1.877841	1.255660	2.808313	.992447	3.553124
.416667	.087765	.049912	1.758406	1.175797	2.629699	.929325	3.327138
.437500	.091341	.059653	1.531198	1.023869	2.289909	.809244	2.897230
.458333	.076655	.064972	.902116	.603221	1.349118	.476773	1.700728
.479167	.056912	.078205	.727736	.486617	1.088330	.384611	1.376972
.500000	.051164	.049253	1.038801	.694617	1.553528	.44410	.965549

TABLE 2
Comparison of Spectra

(a) FREQ	(b) X SPECTRUM	(c) Y SPECTRUM	(d) RATIO	(e) INDIV CI LB	(f) INDIV CI UB	(g) SIMU CI LB	(h) SIMU CI UB
0.000000	.851406	1.079992	.788345	.480849	1.292479	.370523	1.677326
.031250	1.014157	1.208320	.839311	.511936	1.376038	.394477	1.785765
.062500	1.339823	1.366438	.980522	.598067	1.607551	.460846	2.086213
.093750	1.727075	1.436993	1.201867	.733076	1.970443	.564879	2.557159
.125000	1.516686	1.270897	1.193398	.727910	1.956558	.560898	2.539139
.156250	.916654	.841212	1.089683	.664650	1.786519	.512152	2.318470
.187500	.897975	.591308	1.518624	.926281	2.489762	.713755	3.231109
.218750	.809345	.474889	1.704283	1.039523	2.794147	.801015	3.626128
.250000	.443885	.287546	1.543914	.941706	2.531224	.725641	3.284916
.281250	.280113	.182383	1.535845	.936785	2.517994	.721848	3.267748
.312500	.179058	.138035	1.297416	.791355	2.127094	.609786	2.760453
.343750	.173903	.218852	.794613	.484672	1.302756	.373469	1.690663
.375000	.191589	.194449	.985295	.600978	1.615376	.463089	2.096367
.406250	.183002	.151299	1.209537	.737754	1.983018	.568484	2.573478
.437500	.168183	.188995	.889878	.542779	1.458942	.418244	1.893354
.468750	.151892	.181256	.837998	.511135	1.373884	.393860	1.782970
.500000	.125002	.122926	1.016882	.620245	1.667163	.477935	2.163574

The cross correlative nature of the two series can be investigated by decomposing the cross spectrum into squared coherency and phase components. The theoretical coherency and phase spectra for the bivariate linear process are shown in Figure 13. Figures 14 and 15 show the corresponding sample spectra. The sample coherency spectrum in Figure 14, based on a truncation point of $m = 16$, shows reasonably stable behavior and agrees closely with the theoretical spectrum. The most important features of the coherency spectrum is the single large peak and the tendency toward zero at both low and high frequencies. Thus, the majority of the correlation between the two series is centered in the mid-range frequencies. The sample phase spectrum, Figure 15, shows good agreement with the theoretical phase spectrum. The phase spectrum shows that low frequency components of x_t lag those of y_t by approximately 90 degrees but that the phase difference tends to zero at higher frequencies.

An Example Using Real Data - We will now illustrate simulation model validation using time series data from a control variable for a typical missile. Figure 16 presents 800 realizations of simulation data (dashed line) and flight test data (solid line). The visual impression is that both series agree closely. The autocorrelation functions and spectra, shown in Figures 17, 18, 19, and 20 are typical of those associated with AR(2) processes. Each spectra is computed using truncation points of $m = 8, 16$, and 32. There is evidence of a narrow peak in the low frequency range.

Table 3 presents the statistical comparison of the spectra for a truncation point of $m = 32$. The 95 percent simultaneous confidence limits, shown in columns (g) and (h), indicate that at all points in the spectrum containing significant power there is good agreement between the simulation and flight test data. The only simultaneous confidence limits that do not include one are at higher frequencies where the power is low. We conclude that the simulation model and the flight test data are substantially equivalent.

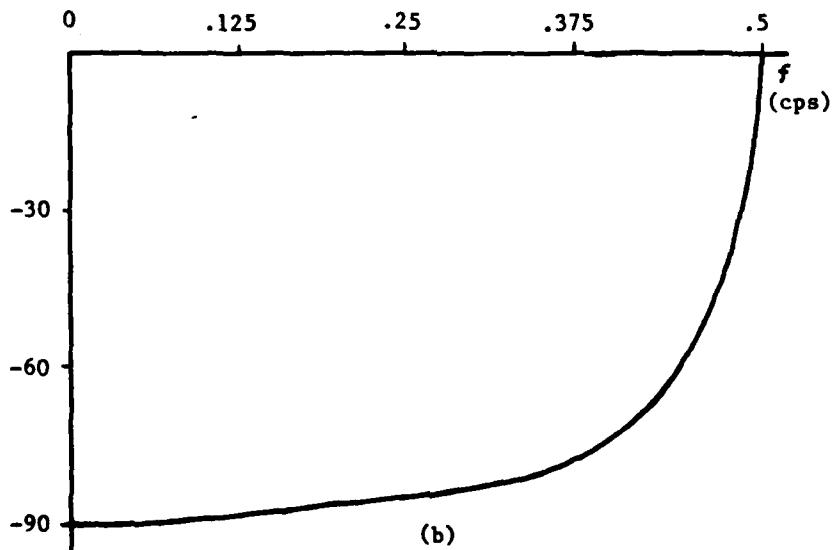
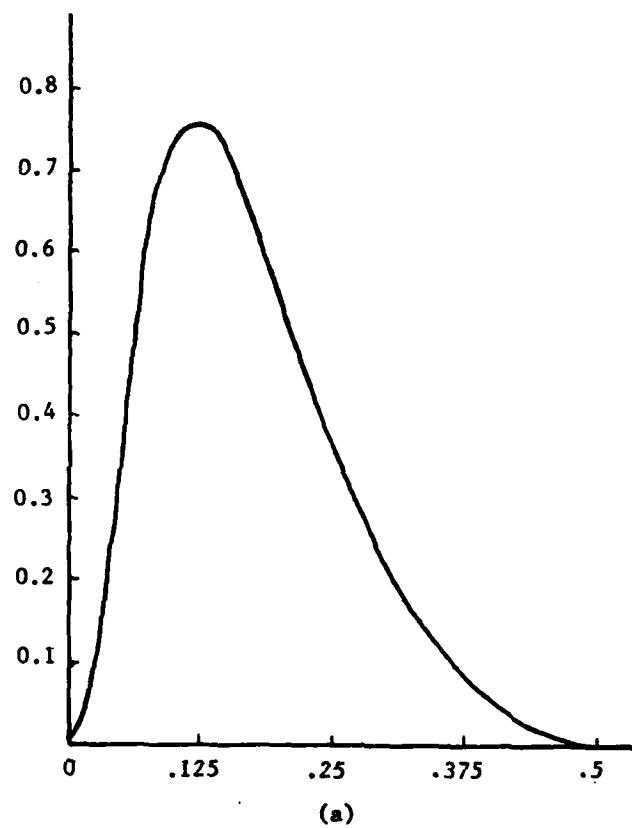


Figure 13. (a) Theoretical Squared Coherency Spectrum and
 (b) Phase Spectrum for the Bivariate Linear Process.

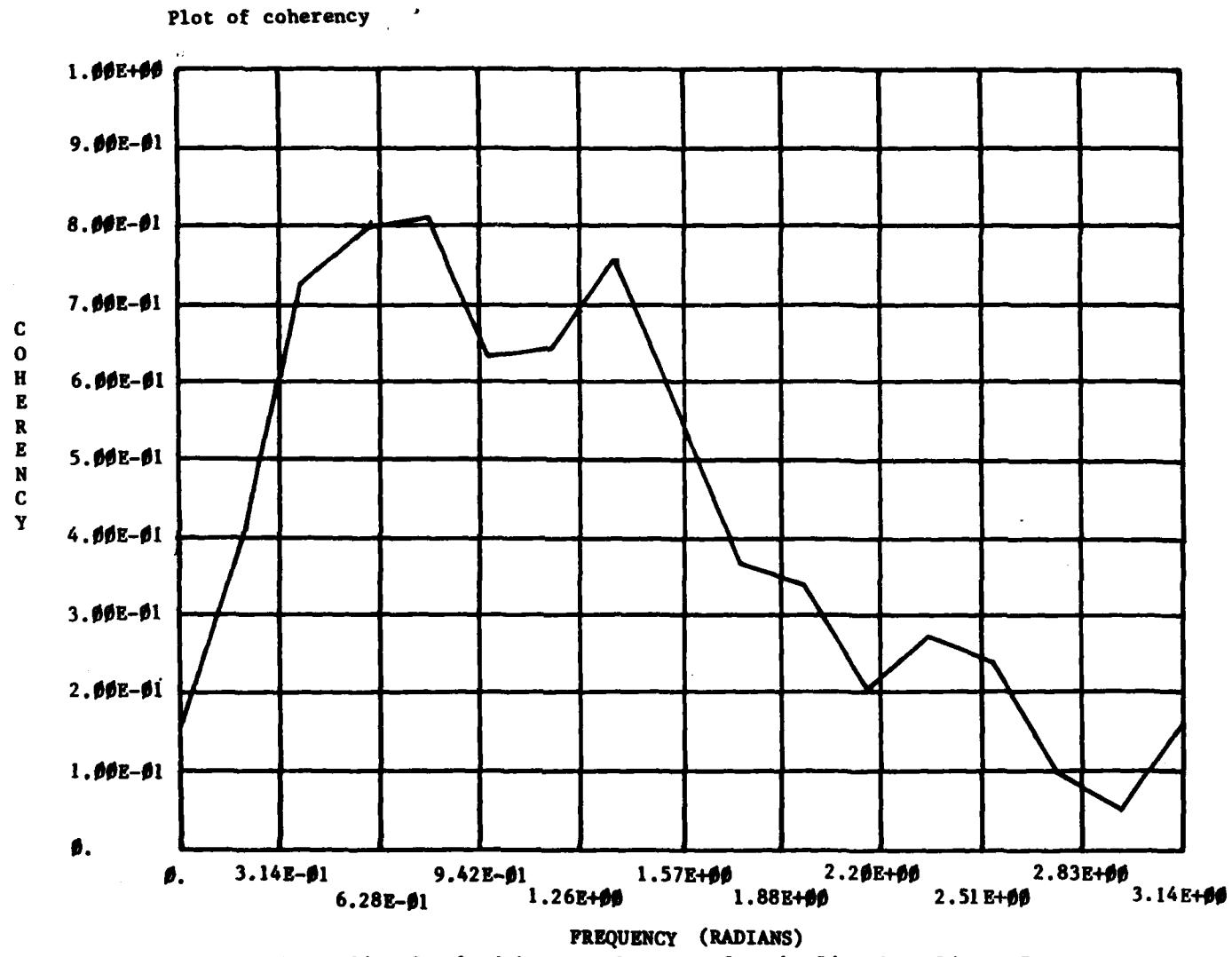


Figure 14. Sample Coherency Spectrum for the Bivariate Linear Process.

Phase of the cross spectrum

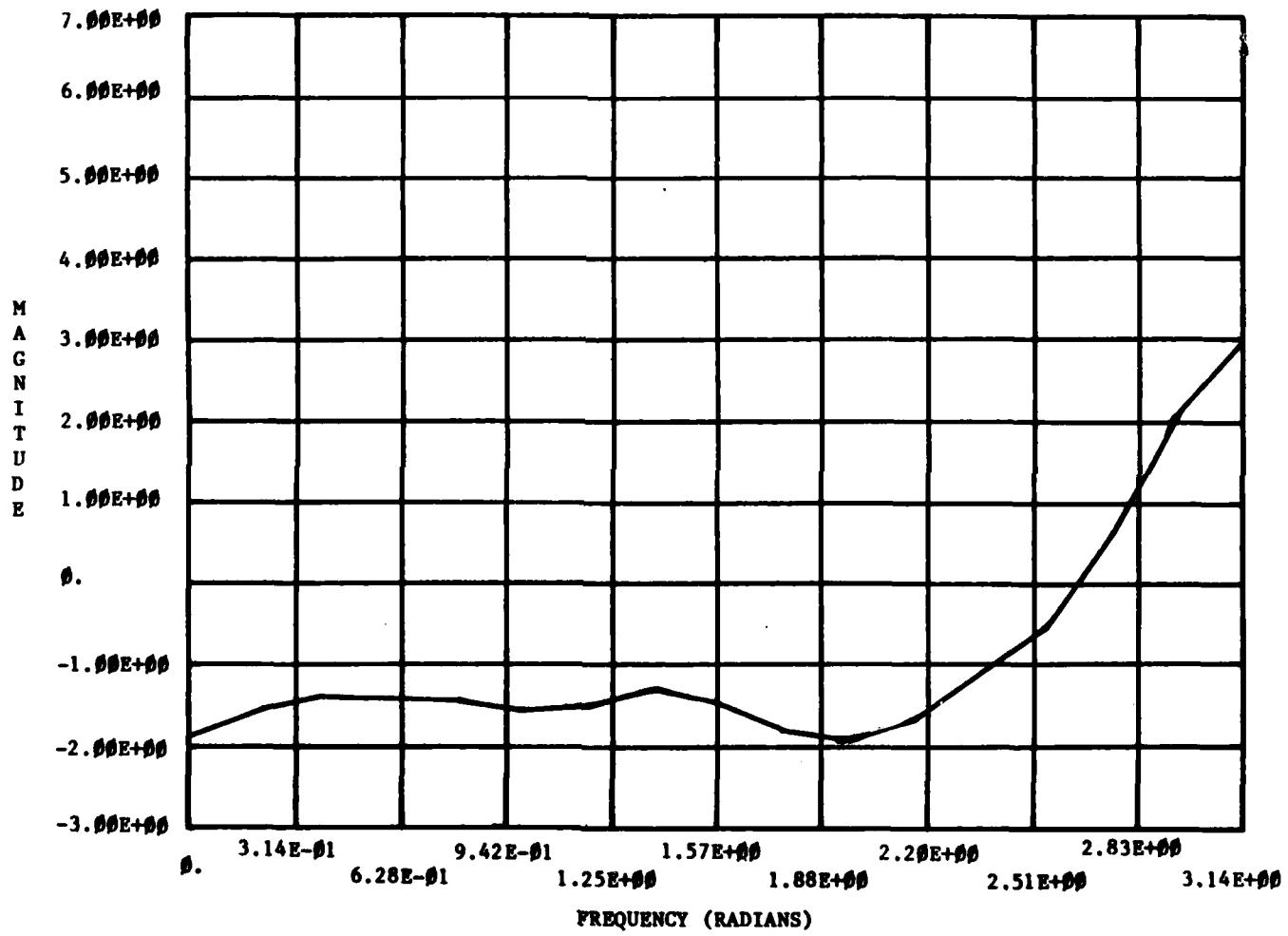


Figure 15. Sample Phase Spectrum for the Bivariate Linear Process.

Plot of original series

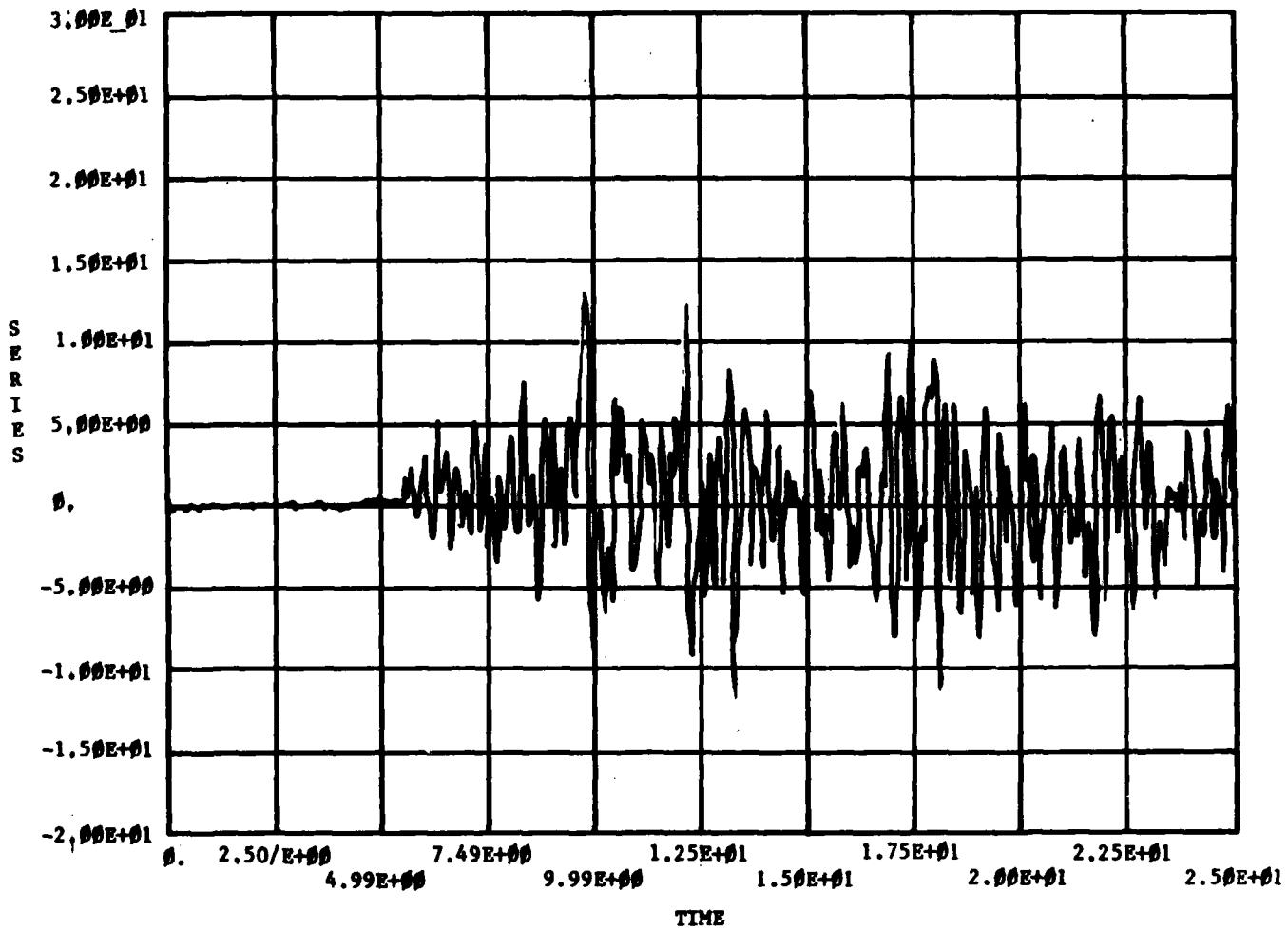


Figure 16. Simulation and Flight Test Data from the Missile

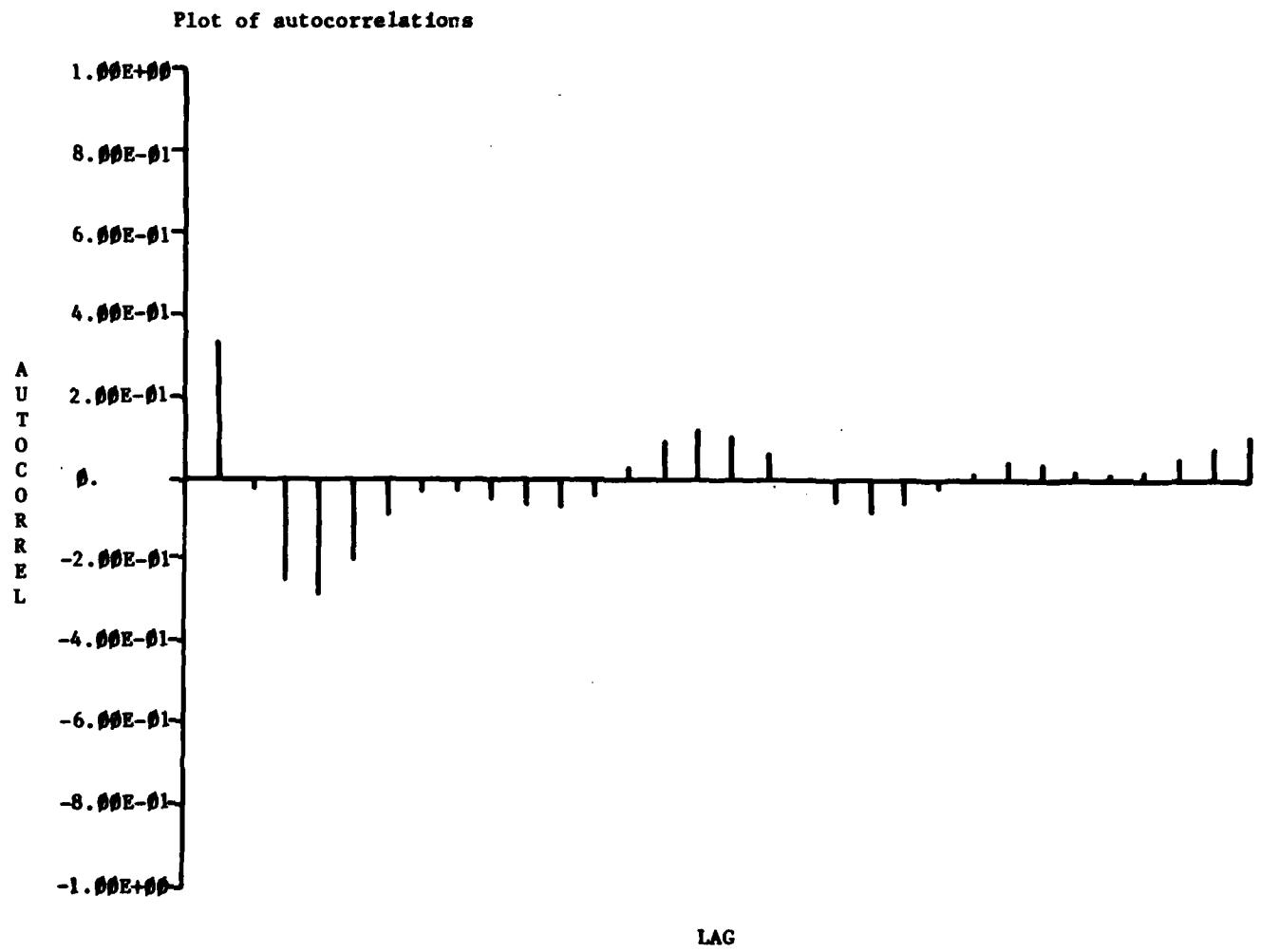


Figure 17. Sample ACF, Simulation Data.

Plot of autocorrelations

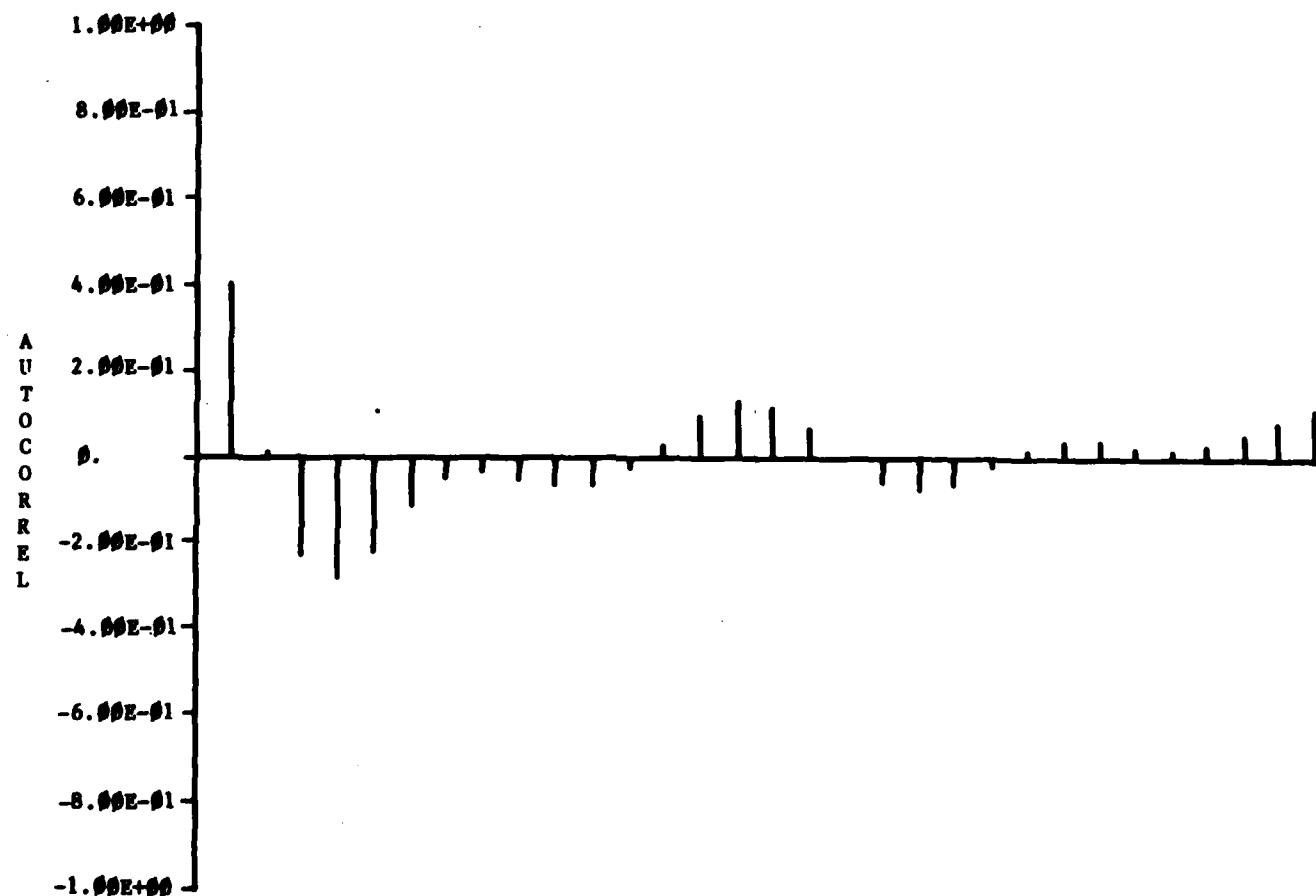


Figure 18. Sample ACF, Flight Test Data

44

Spectrum of series 1 and 2

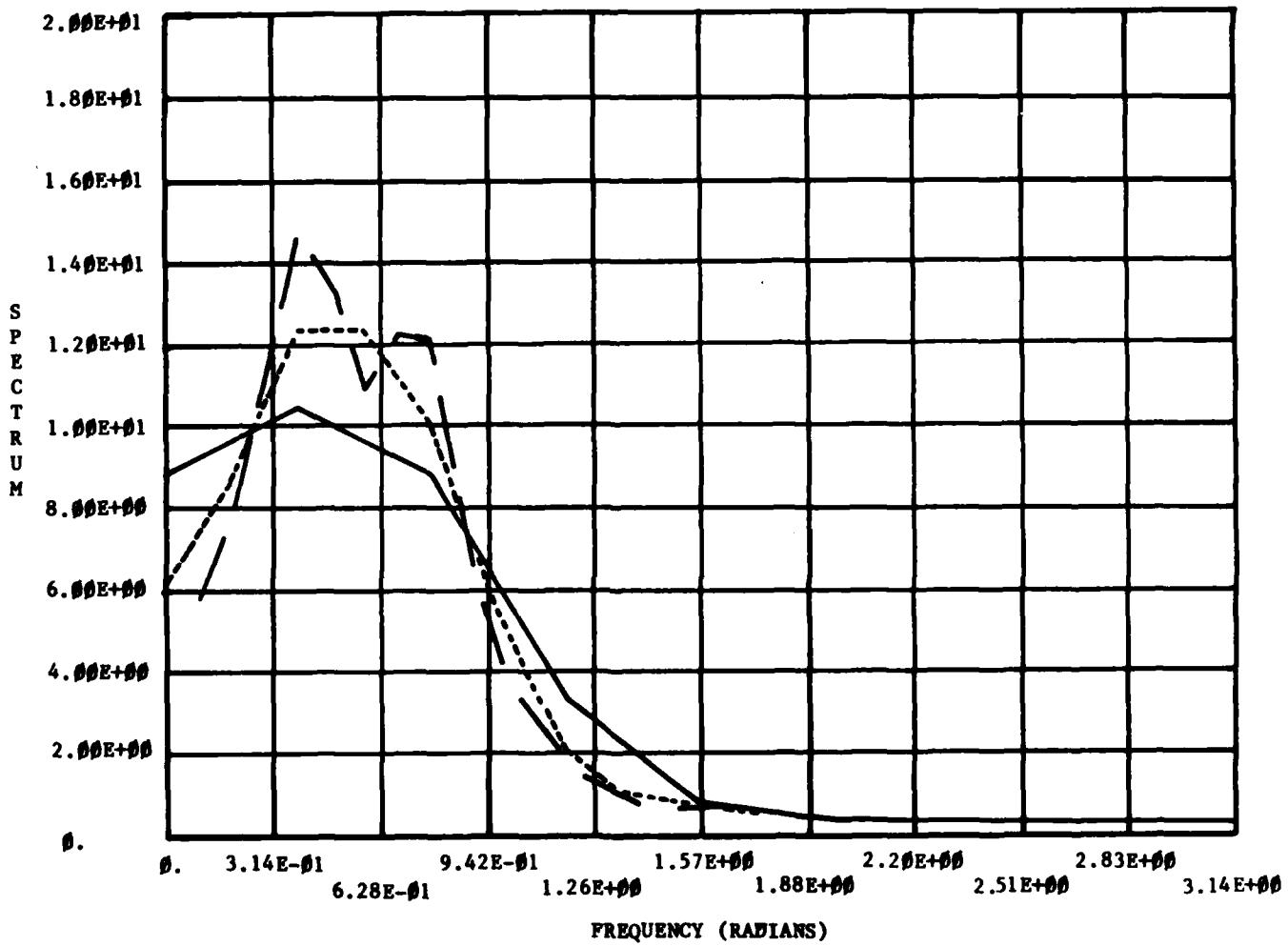


Figure 19. Sample Spectral Estimates, Simulation Data.

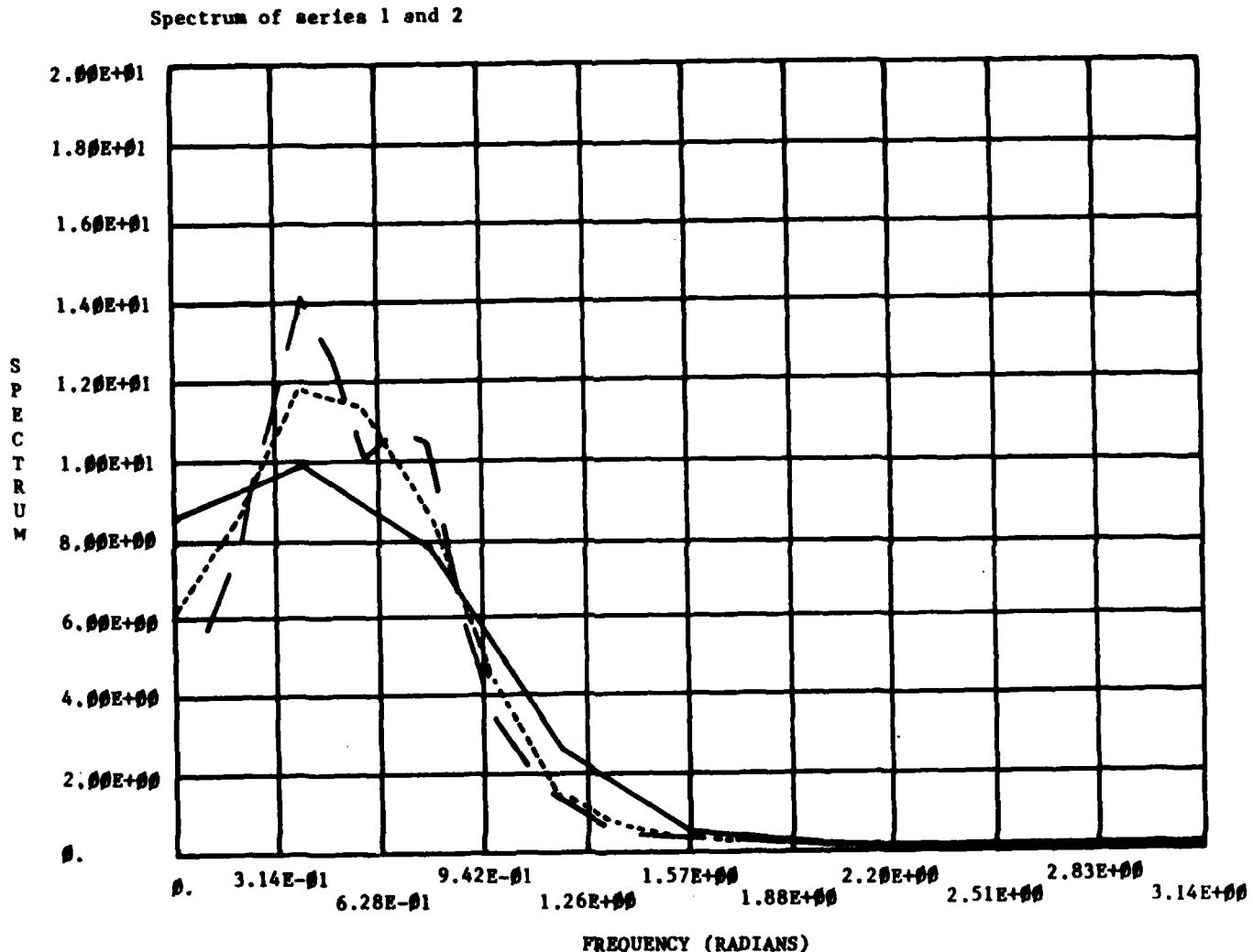


Figure 20. Smoothed Spectral Estimates, Flight Test Data.

TABLE 3
Comparison of Spectra

(a) FREQ	(b) X SPECTRUM	(c) Y SPECTRUM	(d) RATIO	(e) INDIV CI LB	(f) INDIV CI UB	(g) SIMU CI LB	(h) SIMU CI UB
0.000000	5.732432	5.642977	1.015852	.717275	1.438717	.577091	1.788205
.015625	5.737348	5.688931	1.008511	.712091	1.428319	.572920	1.775281
.031250	7.760088	7.678939	1.010568	.713544	1.431233	.574088	1.778902
.046875	11.245462	10.983929	1.023811	.722894	1.449988	.581611	1.802213
.062500	14.743461	14.166088	1.040757	.734860	1.473989	.59123	1.832045
.078125	13.336826	12.569905	1.061012	.749162	1.502676	.602745	1.86770
.093750	10.872416	9.949193	1.092794	.771602	1.547687	.620800	1.923645
.109375	12.208082	10.739831	1.136711	.802611	1.609885	.645748	2.000952
.125000	12.066034	10.281119	1.173611	.828666	1.662145	.666711	2.065907
.140625	7.427049	6.099890	1.217571	.859705	1.724404	.691684	2.143290
.156250	4.282530	3.345625	1.280039	.903812	1.812875	.727171	2.253252
.171875	2.856734	2.104529	1.357422	.958451	1.922471	.771131	2.389470
.187500	1.866200	1.294401	1.441747	1.017992	2.041898	.819035	2.537908
.203125	1.212969	.782643	1.549838	1.094312	2.194982	.880440	2.728179
.218750	.823493	.495703	1.661261	1.172987	2.352788	.943738	2.924318
.234375	.626774	.339631	1.845453	1.303041	2.613653	1.048374	3.248551
.250000	.620323	.307346	2.018323	1.425102	2.858482	1.146579	3.552853
.265625	.693298	.305548	2.269033	1.602123	3.213554	1.289004	3.994178
.281250	.558139	.232748	2.398043	1.693215	3.396266	1.362292	4.221274
.296875	.342962	.124896	2.745978	1.938886	3.889035	1.559949	4.833745
.312500	.287230	.096847	2.965808	2.094104	4.200374	1.684831	5.220712
.328125	.247898	.066223	3.743357	2.643117	5.301589	2.126545	6.589430
.343750	.232529	.057772	4.024955	2.841948	5.700406	2.286517	7.085127
.359375	.187192	.036992	5.060399	3.573057	7.166871	2.874737	8.907819
.375000	.177687	.033696	5.273248	3.723345	7.468321	2.995653	9.282496
.390625	.180469	.019926	9.056866	6.394890	12.826931	5.145070	15.942798
.406250	.152544	.017450	9.314790	6.577006	13.192221	5.291593	16.396823
.421875	.135968	.005899	23.050089	16.275253	32.645058	13.094411	40.575066
.437500	.157999	.010196	15.495962	10.941420	21.946405	8.803025	27.277539
.453125	.121465	.002200	55.201096	38.976502	78.179438	31.358918	97.170475
.468750	.122542	.005398	22.701789	16.029325	32.151774	12.896548	39.961954
.484375	.108422	0.000000	22.701789	16.029325	32.151774	12.896548	39.961954
.500000	.098980	.003867	25.596934	18.073535	36.252068	14.541236	45.058276

APPENDIX
COMPUTER PROGRAMS

The COMP Programs were written in FORTRAN IV extended for implementation on the CDC CYBER 74, utilizing a Tektronix 4014 Graphics Terminal. Routines from the International Mathematical and Statistical Libraries (IMSL) and the Directorate for Management Information Systems (DMIS) General Purpose Computer Subroutines (ALTLIB) augment the standard CDC Library Routines. The programs were designed to be run in the interactive mode with a minimum core requirement of 1100008.

The first program, COMPl, is a preprocessor. Using it, the analyst may determine the "best" set of conditions (smoothing, differencing, window size, etc.) for running the full spectral analysis program. COMPl plots the original series and calculates the autocorrelations. Depending on the interactive commands, one or both of the following options may be selected,

- (1) smooth the data using either a running median scheme or a median/hanning technique
- (2) difference or fit a curvilinear regression model to original or smoothed series.

The spectrum for the original series, the differenced or the residuals from the curvilinear fit is then calculated and plotted.

A hanning window is used for the spectrum calculations. Three spectral window sizes (eg. 8, 16, 32) are allowed. The curvilinear regression routine, as set-up in COMPl, fits a model of the form

$$y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 z_t + a_t$$

where

$$z_t = \begin{cases} 0 & t \leq t^* \\ t - t^* & t > t^* \end{cases}$$

and t^* is the t value at which a change in the curve is observed.

This program calculates the Durbin-Watson statistic, the basis of a test of autocorrelation in regression analysis (see Draper and Smith) (4). In addition, the value of Theil's Inequality Coefficient (TIC) along with Measures for use in determining if the difference between the two series is a result of a time lag, scale or bias error are provided.

The second and third programs, COMP2 and COMP3, are run, using the experience gained from running COMP1, for the full spectral analysis. COMP3 makes the same calculations as COMP1 and calculates both the $100(1-\alpha)$ percent individual and simultaneous confidence intervals on the ratio of the true spectra, the coherency function, the cross spectra, phase and gain. This program is particularly useful when comparing series for some causal relationship. If there is no interest in the interrelationship between the two series one may run COMP2. COMP2 does not output the cross spectral results.

Input to these programs is from the disc (TAPE1, TAPE 2, TAPE 3) and the console. TAPE 1 and TAPE 2 are data files. TAPE 3 contains the labels to be used with the graphics, the format for reading the data files, etc. The following is an example of the structure of TAPE 3:

0 (F14.8)	0.		
LAG	AUTOCORRELATION SERIES	FREQUENCY TIME	SPECTRUM COHERENCY
MAGNITUDE		PLOT OF ORIGINAL SERIES	
PLOT OF AUTOCORRELATIONS		SPECTRUM OF SERIES 1 AND 2	
PLOT OF DIFF/DETREND SERIES		PHASE OF THE CROSS SPECTRUM	
CROSS SPECTRUM OF THE SERIES		PLOT OF COHERENCY	
PLOT OF GAIN			
RECORD 1			
ISKIP, XSTAR		I10, F10.0	
RECORD 2			
DFORM		8A10	
RECORD 3			
AL1, AL2, AL3, AL4		8A10	
RECORD 4			
AL5, AL6, AL7, AL8		8A10	
RECORD 5			
TITL1, TITL2		6A10	
RECORD 6			
TITL3, TITL4		6A10	
RECORD 7			
TITL5, TITL6		6A10	
RECORD 8			
TITL7, TITL8		6A10	

LIST OF INPUT VARIABLE NAMES

ISKIP - Number of RECORDS to skip on the data files

XSTAR - Regression breakpoint

DFORM - Format for reading the data files

NV - Number of variables in the data for each series, NV is L.E.2

ISIZE - Number of observations for each variable ISIZE is L.E. 250

X,Y - NV x 250 data arrays

MS - Number of the variable to compare

ISM - Smoothing indicator, 1 = YES, 0 = NO

ICNT - Type smoothing, 1 = MEDIAN, 2 = MED/HAMMING

K - Number of autocorrelations to compute or number of lags
for FTFREQ

ID - Difference/spectra plot/analysis complete indicator

AL1,AL2,...,AL8 - Labels for ABSISSA and ordinate of plots

TITL1,...,TITL8 - Titles for plots

Output is both printed format on file output and special graphics.

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